Wireless Channels
Path Loss and Shadowing

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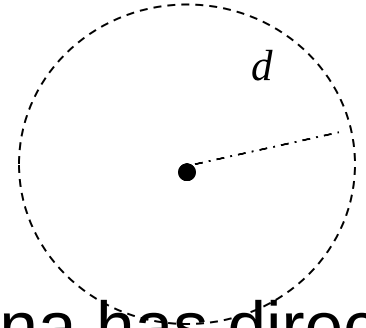
- Wireless channel susceptible to
  - Noise
  - Interference
  - Channel impediments
- Impediments change over time unpredictably due to
  - User movement
  - Environment dynamics
- Channel impediments
  - Path loss and shadowing (~deterministic, large scale)
  - Multipath (~statistical, small scale)

Path Loss and Shadowing

- Free space propagation, line of sight (LOS) attenuation
- An isotropic tx antenna with power P_t *Watts*
  - Power density at distance *d*

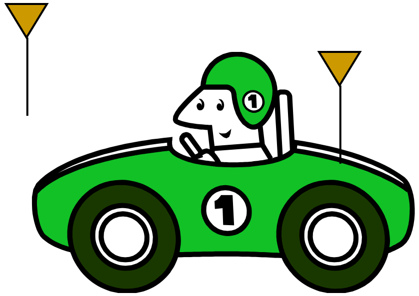


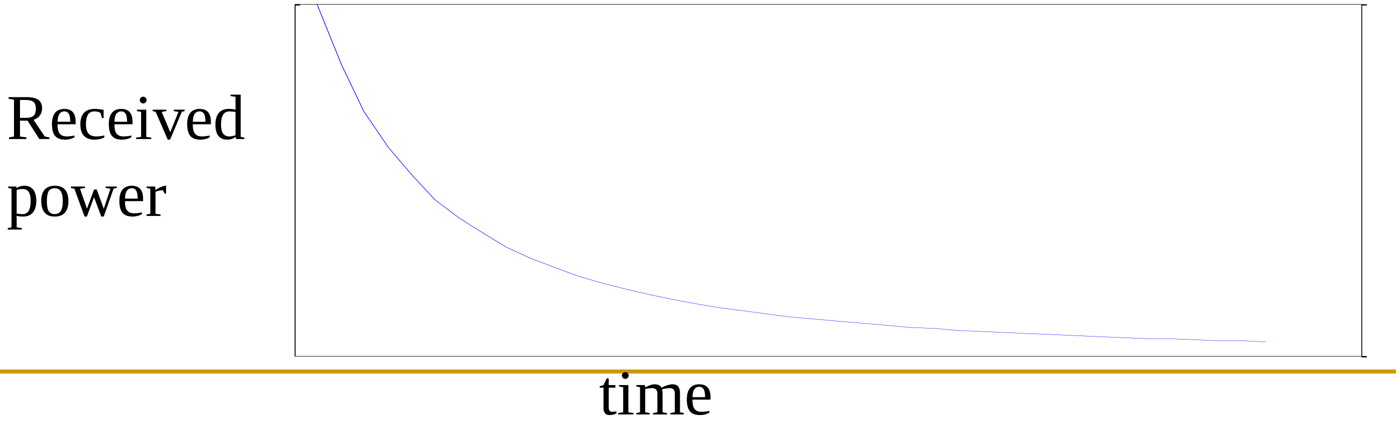
W/m^2

- If tx antenna has directivity (field radiation pattern)
- Rx antenna gathers a portion of the radiated power proportional to its cross-sectional area.

$$G_t \frac{P_t}{4\pi d^2} A_r$$

- From EMT $A_r = G_r \frac{\lambda^2}{4\pi}$
- Friis transmission formula





- Remarks
  - Gains depend on antenna physical properties.
  - Other losses (atmosperic absorption) sometimes effective
  - Path loss

$$P_r = P_t G_t G_r / P_L$$

  - Usually in dB

$$(P_r)_{dB} = (P_t)_{dB} + (G_t)_{dB} + (G_r)_{dB} - (P_L)_{dB} \qquad \left(-(P_a)_{dB}\right)$$

  - Used directly for satellite communications and radio links

- Transmitted signal

$$\begin{aligned} s(t) &= \Re\left\{u(t)e^{j(2\pi f_c t+\phi_0)}\right\} \\ &= \\ &= \end{aligned}$$

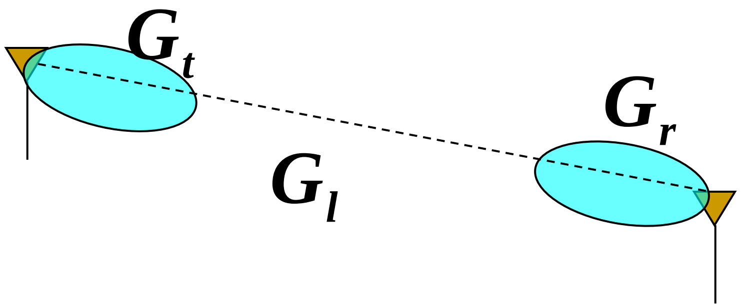
- Equivalent lowpass representation of bandpass signals
  - $\boldsymbol{u(t)}$ complex envelope, equivalent lowpass signal
- Received signal $r(t) = \Re\left\{v(t)e^{j(2\pi f_c t+\phi_0)}\right\},$

$$P_r = P_t G_t G_r \frac{c^2}{(4\pi f d)^2}$$

- With free space path loss

$$r(t) = \mathrm{Re}\{\frac{c\sqrt{G_l}}{4\pi f d} u(t-\tau) e^{j2\pi f(t-\tau)}\}$$

  - Link gain comprised of transmit and receive antenna gains

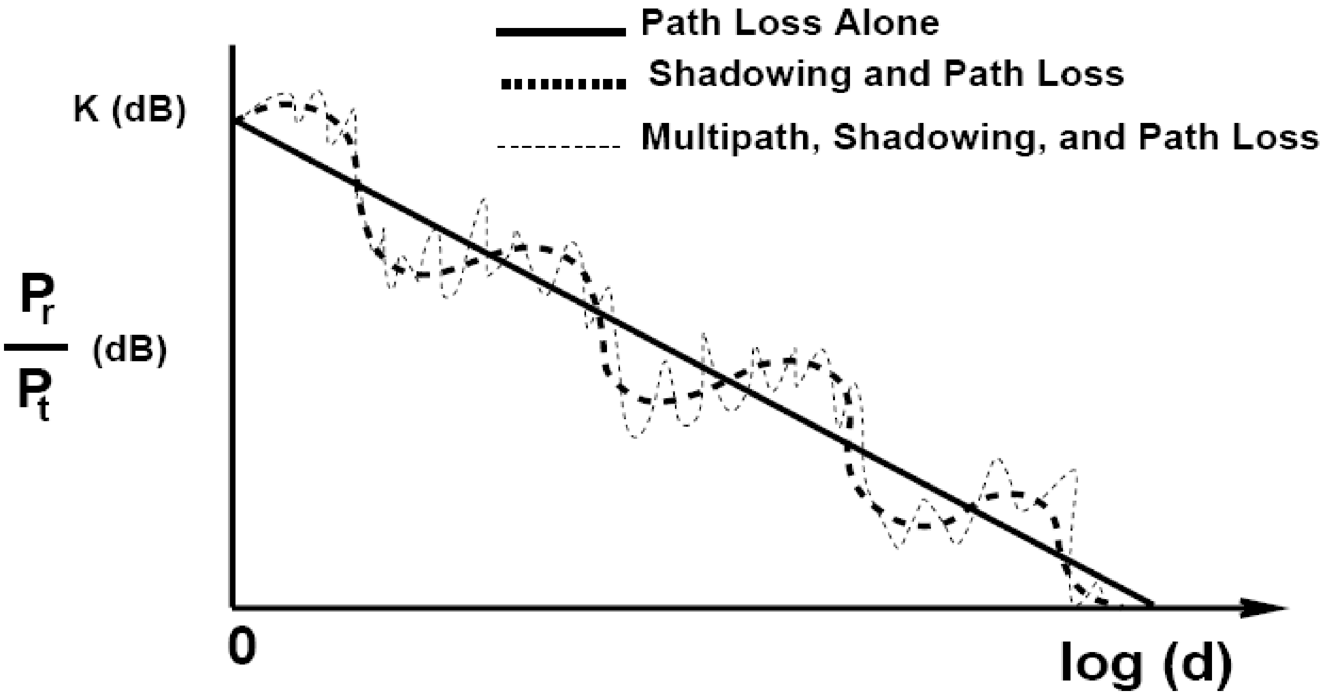


  - Delay due to the distance traveled by the EM waves

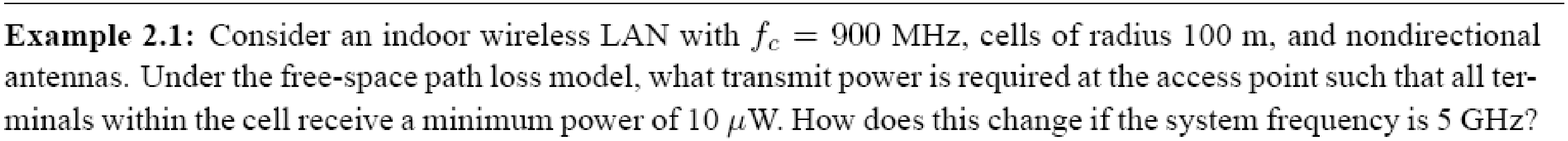
- Path loss (usually antenna gains excluded)

$$P_L = \frac{G_l P_t}{P_r} = \left(\frac{c}{4\pi f d} \right)^{-2}$$

$$P_{L,dB} = -20 \log_{10} \left(\frac{c}{4\pi f d} \right)$$

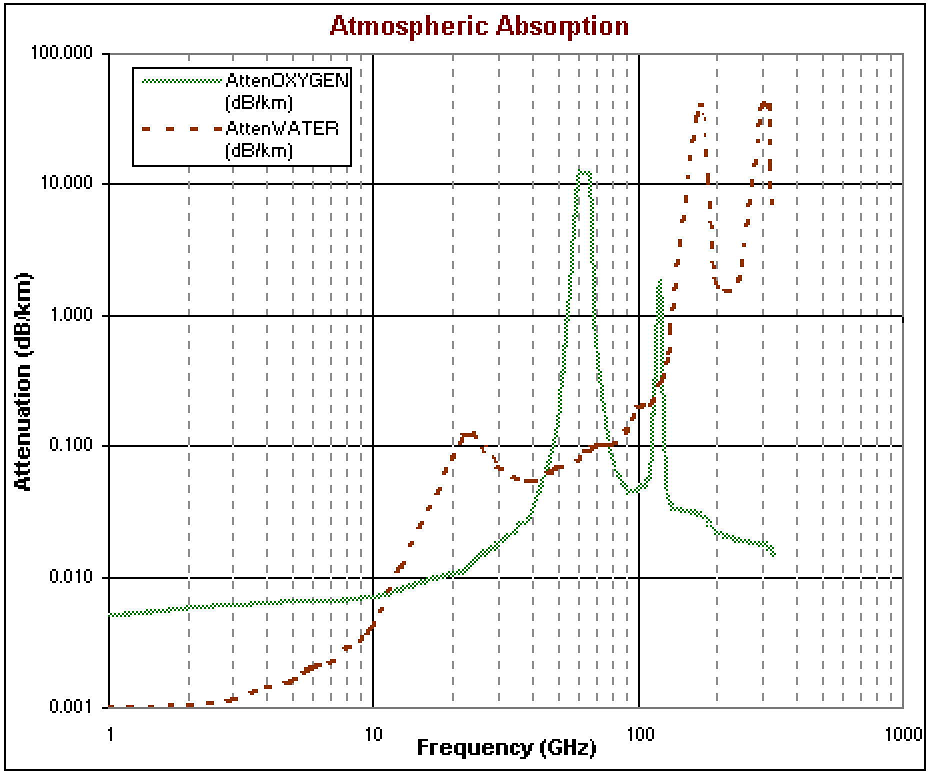


- Example

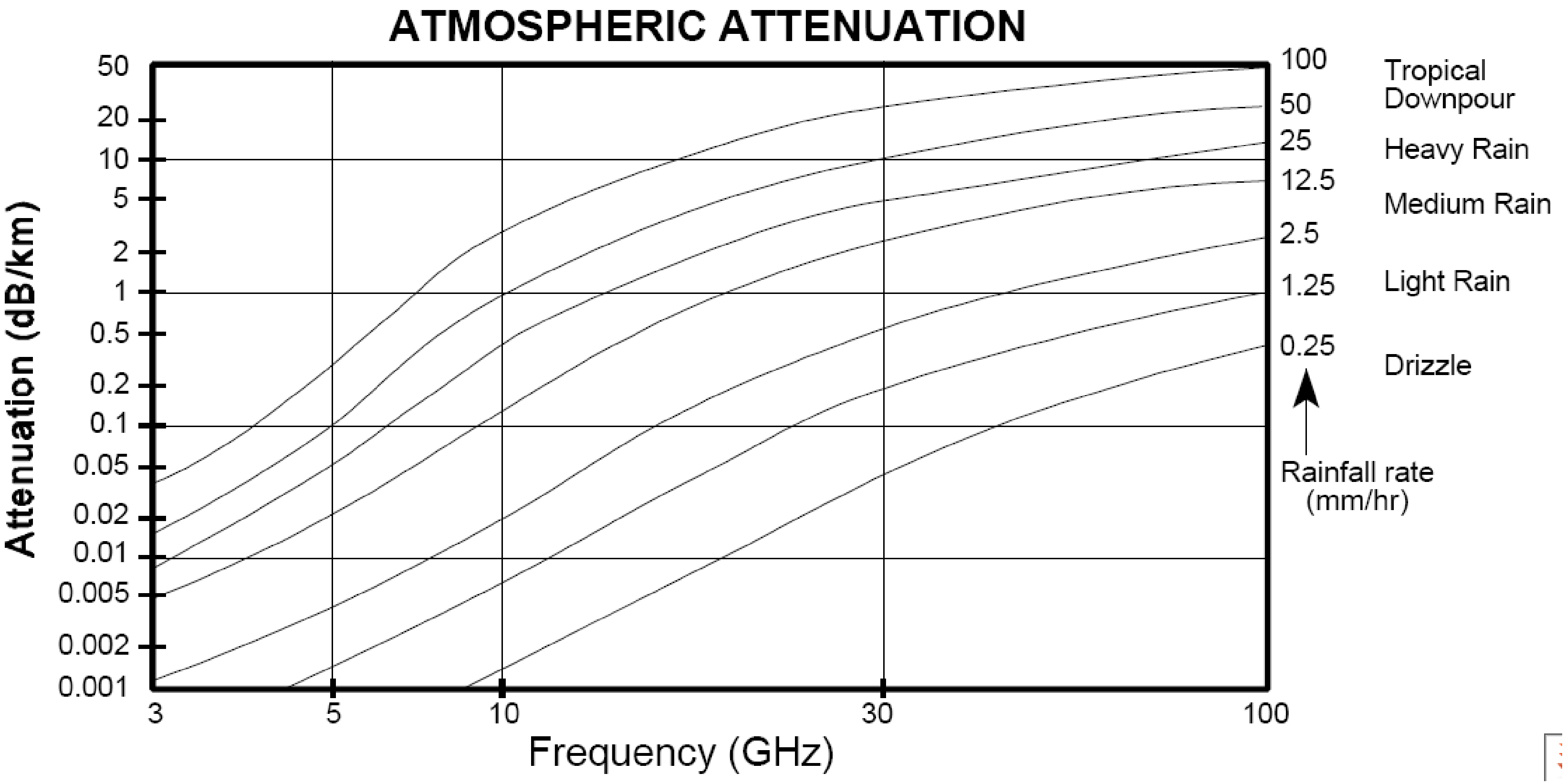


Example 2.1: Consider an indoor wireless LAN with $f_c = 900$ MHz, cells of radius 100 m, and nondirectional antennas. Under the free-space path loss model, what transmit power is required at the access point such that all terminals within the cell receive a minimum power of 10 μW. How does this change if the system frequency is 5 GHz?

- Atmospheric attenuation
  - Oxygen, water
  - Rain, fog
  - Height dependent

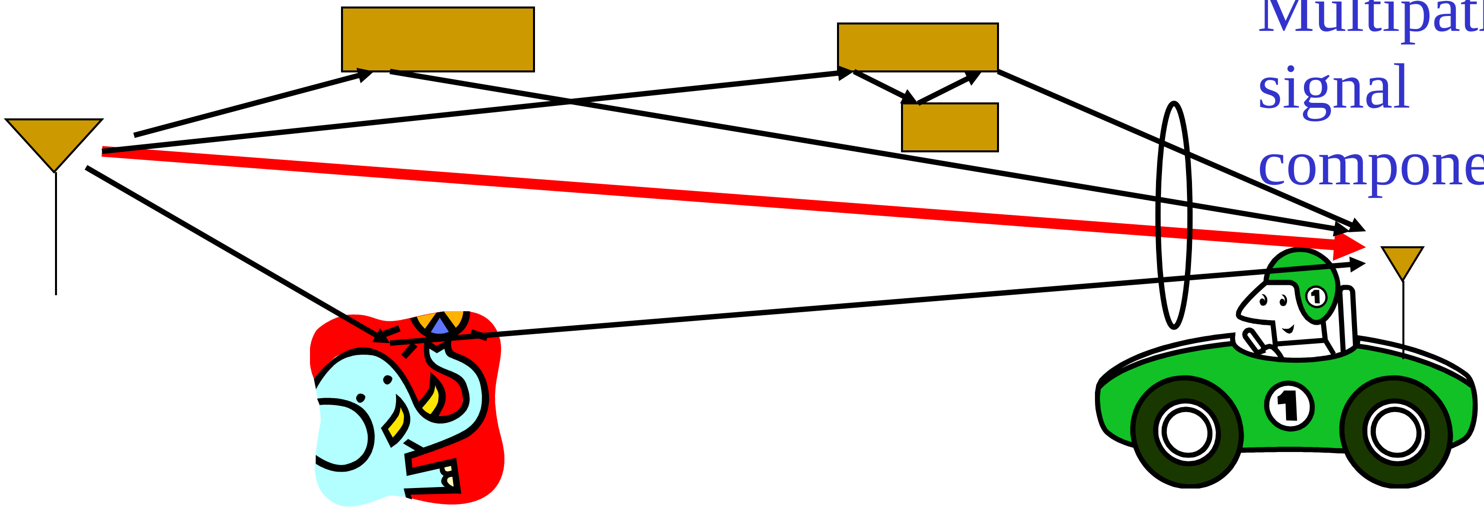


http://www.rfcafe.com/references/electrical/atm_absorption.htm



http://www.tscm.com

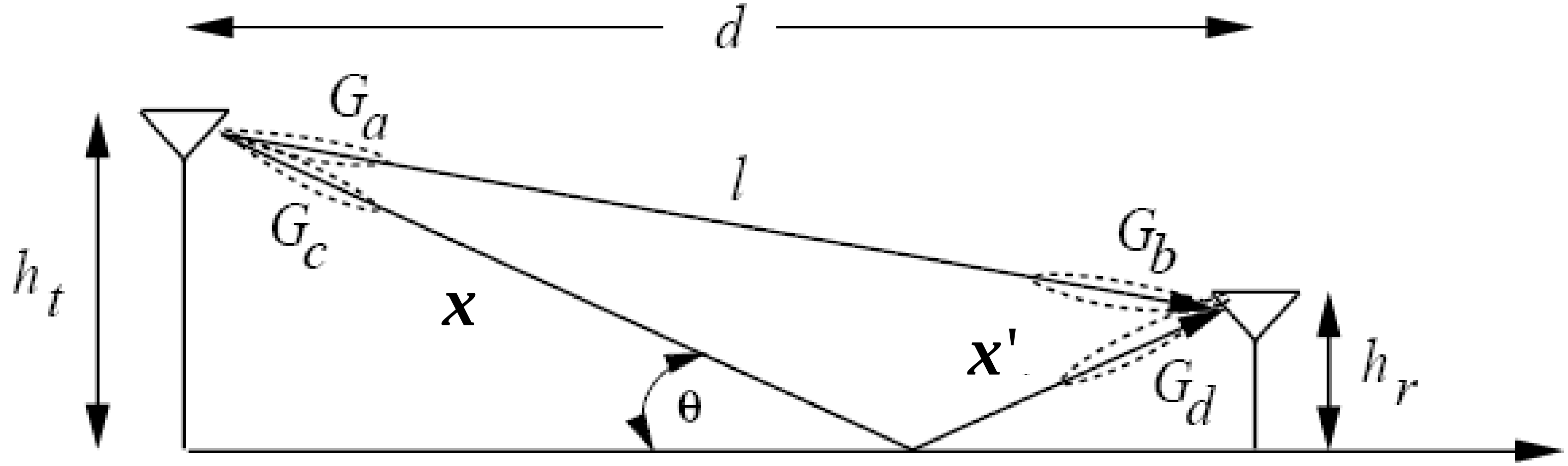
Ray Tracing



Multipath signal components

- Many objects in surroundings
  - Reflection (EMW on an object larger than wavelength)
  - Diffraction (path obstructed by a surface with sharp irregularities)
  - Scattering (medium densely consists of objects smaller than wavelength)

- Solve Maxwell's equations with boundary conditions
- Too complex
- Everything should be perfectly known
- Simplification necessary
- Ray tracing
  - Assume a finite number of reflectors with known location and dielectric properties



Ground reflection coefficient

$$r(t) = \Re\left\{ \frac{\lambda}{4\pi} \left[\frac{\sqrt{G_l}\, u(t-\kappa) e^{-j2\pi l/\lambda}}{l} + \frac{R\sqrt{G_r}\, u(t-\tau) e^{-j2\pi(x+x')/\lambda}}{x+x'} \right] e^{j2\pi f_c t} \right\}$$

- If transmitted signal is slowly changing in relation to τ , $u(t-\tau)\approx u(t), u(t-\kappa)\approx u(t)$

$$P_r = P_t\left(\frac{\lambda}{4\pi}\right)^2\left|\frac{\sqrt{G_l}}{l}+\frac{R\sqrt{G_r}\,e^{-j\Delta\phi}}{x+x'}\right|^2, \quad \Delta\phi = 2\pi(x+x'-l)/\lambda$$

$$x+x'-l=\sqrt{(h_t+h_r)^2+d^2}-\sqrt{(h_t-h_r)^2+d^2}$$

- Asymptotic case
  - d is large $x + x' \approx l \approx d, \quad \theta \approx 0$
  - $G_l \approx G_r$
  - For earth and road surfaces $R \approx -1$

$$P_r \approx \left[\frac{\lambda\sqrt{G_l}}{4\pi d}\right]^2 \left[\frac{4\pi h_t h_r}{\lambda d}\right]^2 P_t =$$

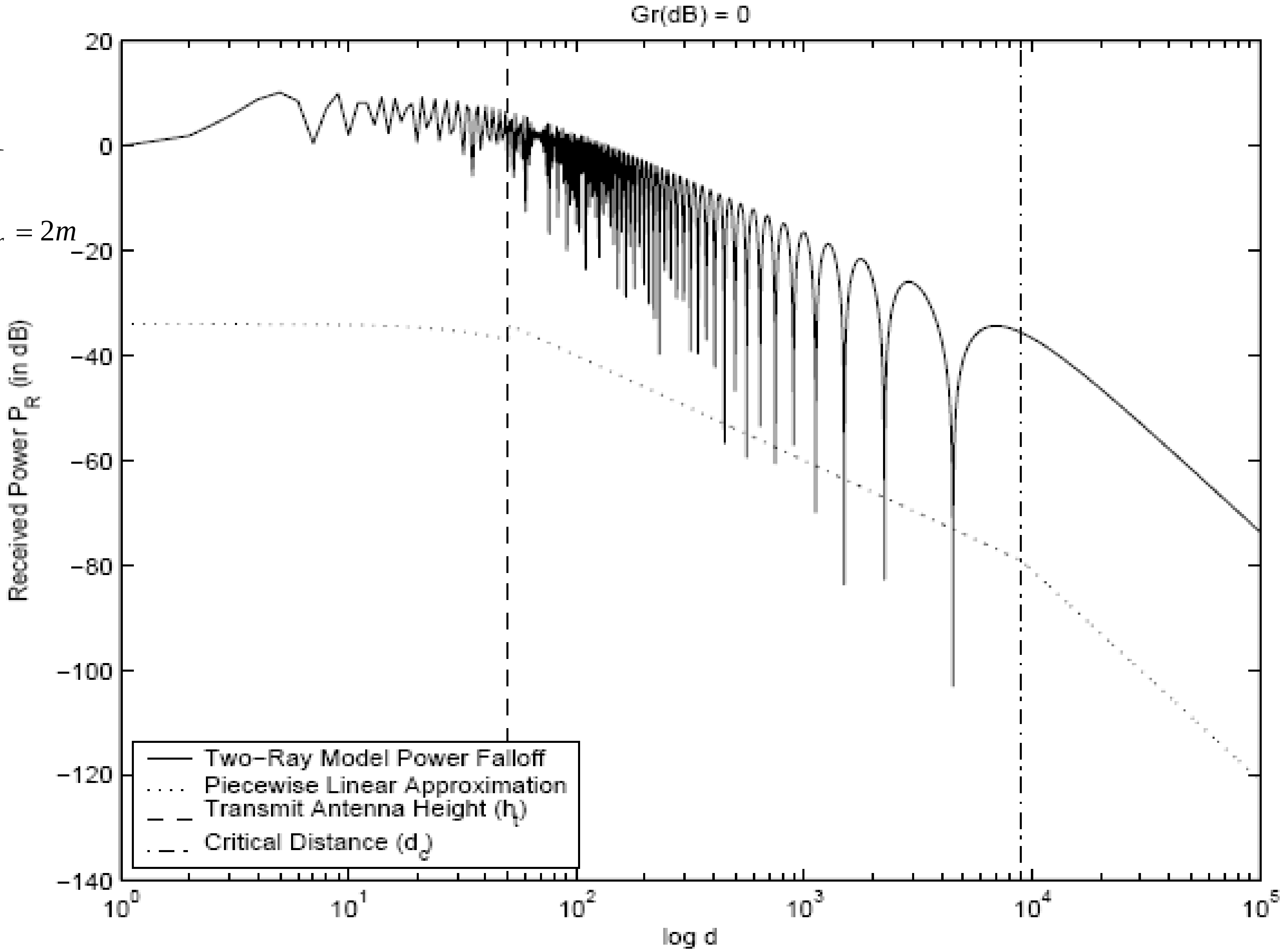
  - Receiver power falls off as d^{-4}
  - Independent of frequency since combination of two rays effectively forms an antenna array (antenna array gain does not necessarily decrease with frequency)

$900 MHz$

$G_l = G_r = 1$

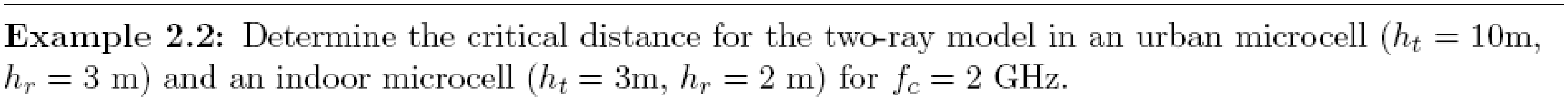
$R = -1$

$h_t = 50m, h_r = 2m$



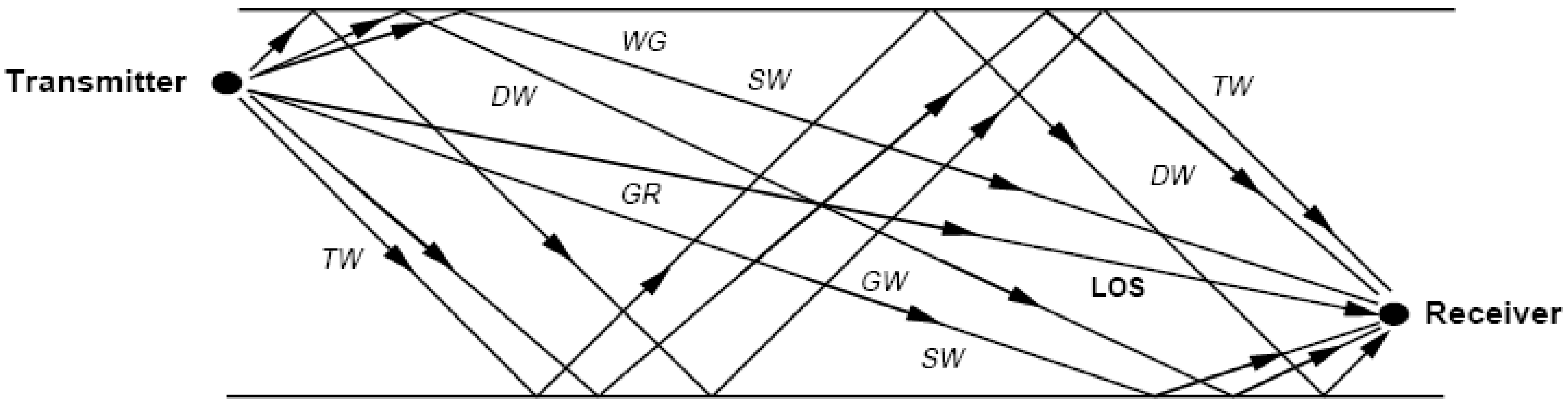
$$d_c = 4h_t h_r / \lambda.$$

- Example



Example 2.2: Determine the critical distance for the two-ray model in an urban microcell ($h_t = 10$m, $h_r = 3$ m) and an indoor microcell ($h_t = 3$m, $h_r = 2$ m) for $f_c = 2$ GHz.

- Ten-ray model
  - For urban microcells
  - Flat city with 90 degrees intersecting linear streets (rectilinear streets)
  - Buildings along both sides of streets
  - Building-lined streets act as dielectric canyon to the propagating signal.
  - Since signal energy is dissipated with each reflection, more than 3 reflections can be generally ignored.



$$r_{10ray}(t) = \Re\left\{ \frac{\lambda}{4\pi} \left[\frac{\sqrt{G_l}u(t)e^{j(2\pi l)/\lambda}}{l} + \sum_{i=1}^{9} \frac{R_i\sqrt{G_{x_i}}u(t-\tau_i)e^{j(2\pi x_i)/\lambda}}{x_i} \right]^{*} e^{j(2\pi f_c t+\phi_0)}, \right\}$$

- Typical power falloff $\propto d^{-2}$
- Some empirical studies obtained power falloff proportional to

- Generalized ray tracing
  - Diffracted and scattered rays also taken into account
  - Leads to a complicated path loss model.
  - Simplifications needed
- Local mean received power (LMRP)
  - Ray tracing depends on exact tx/rx locations (phase)
  - Only a mean received power usually required for link quality
  - Cellular systems utilize LMRP for power control and handoff

Empirical Path-Loss Models

- Most wireless systems operate in complex propagation environments
  - Cannot be accurately modeled by free space prop. or ray tracing
- Path-loss models developed over the years to predict path loss in typical wireless environments
  - Large urban macrocells
  - Urban microcells
  - Inside buildings, …
- Never forget: these are just models!

- These models based on empirical measurements
  - Over a given distance
  - In a given frequency range
  - For a particular geographical area or building
- One must be careful in using these models for other scenarios.
- Path loss, shadowing, multipath all contribute to received power in empirical measurements
- Averaging to remove multipath effects
  - Local mean attenuation over several λ
  - Repetitions throughout the environment
  - Repetitions in similar environments

Okumura Model

- Large urban macrocells, 1-100kms
- Base station-to-mobile measurements in Tokyo

$$P_L(d)\ \text{dB} = L(f_c, d) + A_{mu}(f_c, d) - G(h_t) - G(h_r) - G_{AREA}$$

Path loss

Median attenuation

Antenna height gains

Gain due to type of environment

- Empirical formulas

$$G(h_t) = 20\log_{10}(h_t/200), \quad 30m < h_t < 1000m$$

$$G(h_r) = \begin{cases} 10\log_{10}(h_r/3) & h_r \leq 3m \\ 20\log_{10}(h_r/3) & 3m < h_r < 10m \end{cases}.$$

- Others obtained from Okumura's empirical plots
- Corrections proposed later

- Hata model
  - Closed-form formula for Okumura’s model
    - Frequency in MHz, distance in km
  - Correction factor for the mobile antenna height based on the size of the coverage area
    - Small-to-medium size cities

$$a(h_r) = (1.1\log_{10}(f_c) - .7)h_r - (1.56\log_{10}(f_c) - .8)\mathrm{dB}$$

    - Larger cities $f_c > 300\mathrm{MHz}$

$$a(h_r) = 3.2(\log_{10}(11.75h_r))^2 - 4.97\ \mathrm{dB}$$

    - Othe

$$P_{L,suburban}(d) = P_{L,urban}(d) - 2[\log_{10}(f_c/28)]^2 - 5.4$$

$$P_{L,rural}(d) = P_{L,urban}(d) - 4.78[\log_{10}(f_c)]^2 + 18.33\log_{10}(f_c) - K$$

    - K ranges from 39.54(countryside) to 40.94(desert)

  - Hata well approximated Okumura for $d > 1\text{km}$
  - Hata does not model well the current cellular systems with smaller cell sizes and higher frequencies, indoor environments
- COST 231 Extension to Hata
  - $30\text{m} < \text{h}_\text{t} < 200\text{m}, 1\text{m} < \text{h}_\text{r} < 10\text{m}$

$$P_{L,urban}(d)\text{dB} = 46.3 + 33.9\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d) + C_M$$

  - C_M 0dB for medium-sized cities, suburbs; 3dB for metropolitan areas
- Piecewise Linear (Multislope) Model
  - Empirical measurements are fitted to piecewise linear functions

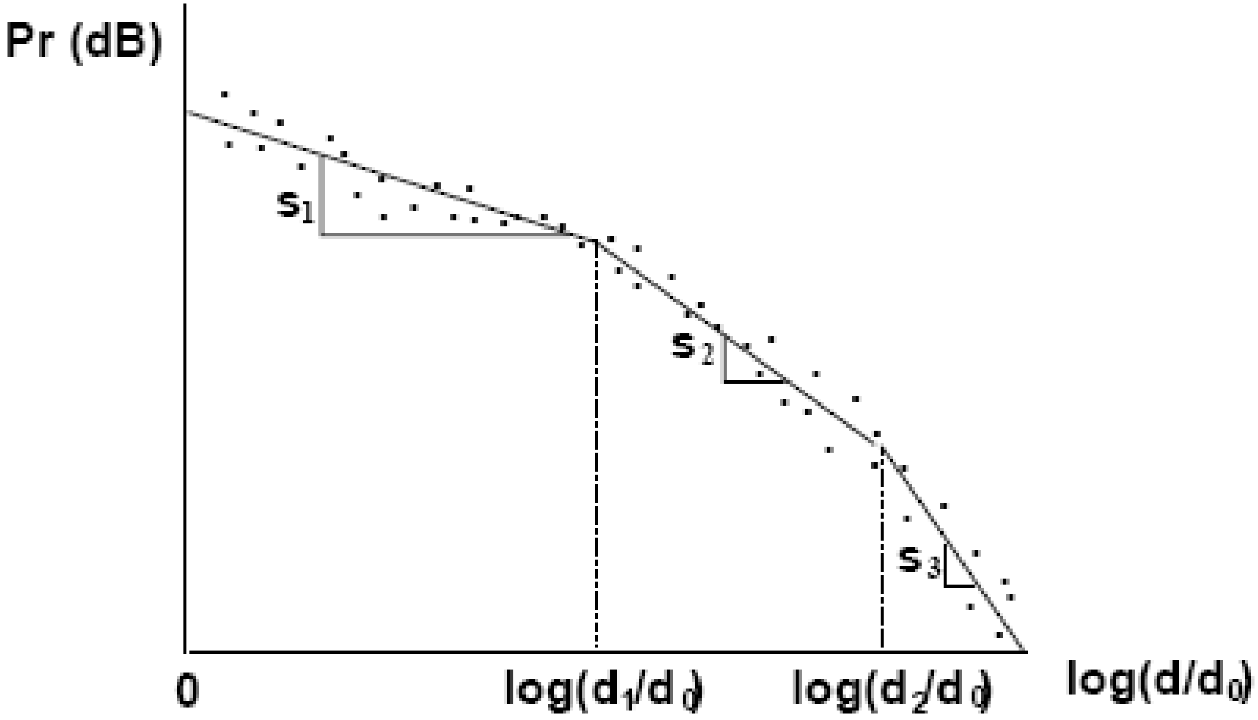
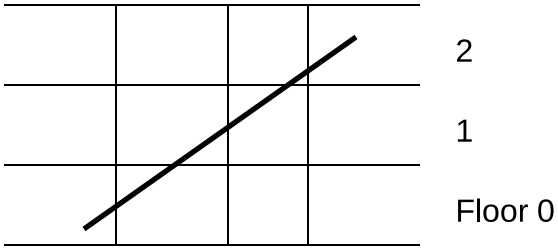
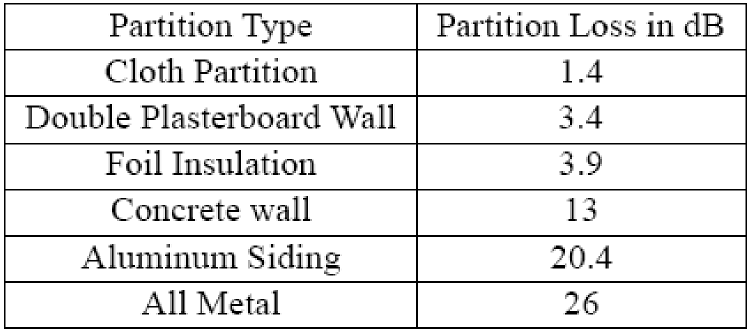


Figure 2.9: Piecewise Linear Model for Path Loss.

- Indoor Attenuation Factors
  - Penetration through
    - Walls
    - Floors
    - Objects
    - Glass, …
  - All these factors significantly affect indoor path loss
  - Floors
    - Depends on building material
    - Attenuation largest for the 1st passed floor (10-20dB)
    - Decreases with subsequent floors (6-10dB, a few dB for larger than 4 floors)
    - Rappaport has details



- Partition losses (walls)



Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

Table 2.1: Typical Partition Losses

- Losses by different studies vary widely
- Very hard to make generalizations

$$P_r \text{ dBm} = P_t \text{ dBm} - P_L(d) - \sum_{i=1}^{N_f} FAF_i - \sum_{i=1}^{N_p} PAF_i$$

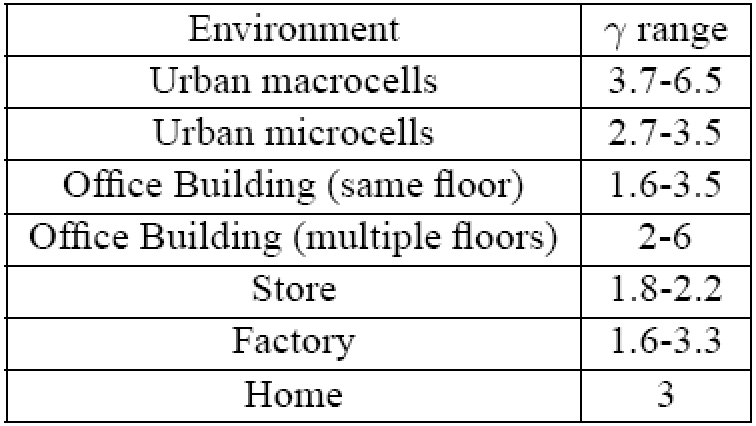
- FAF: floor attenuation factor
- PAF: partition attenuation factor

Simplified Path-Loss Model

- Simplified models necessary for system design

$$P_r = P_t K \left[\frac{d_0}{d}\right]^{\gamma}$$

- K unitless constant depending on antenna characteristics and average channel attenuation
- d_0 reference distance for antenna far field
- γ path-loss coefficient

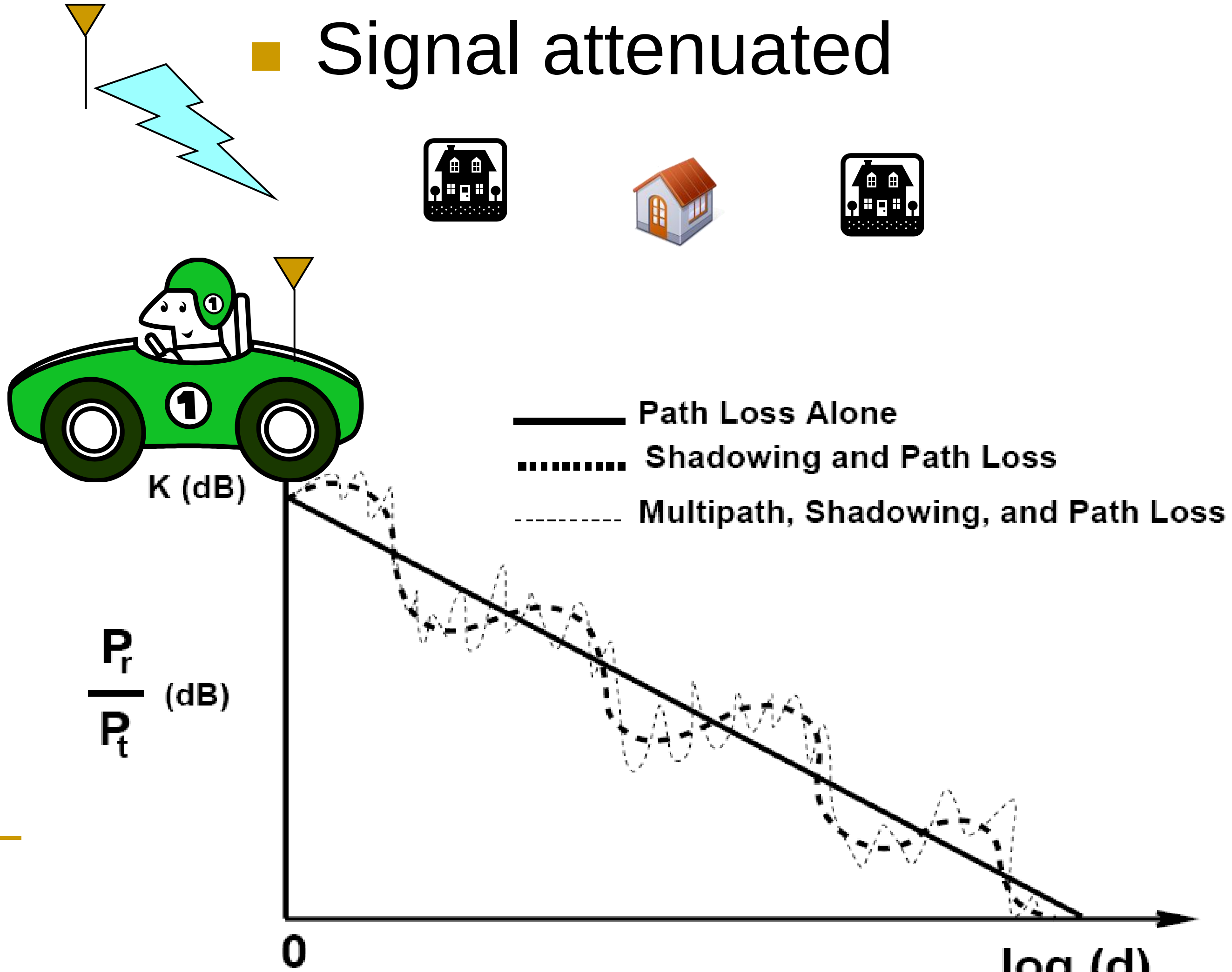


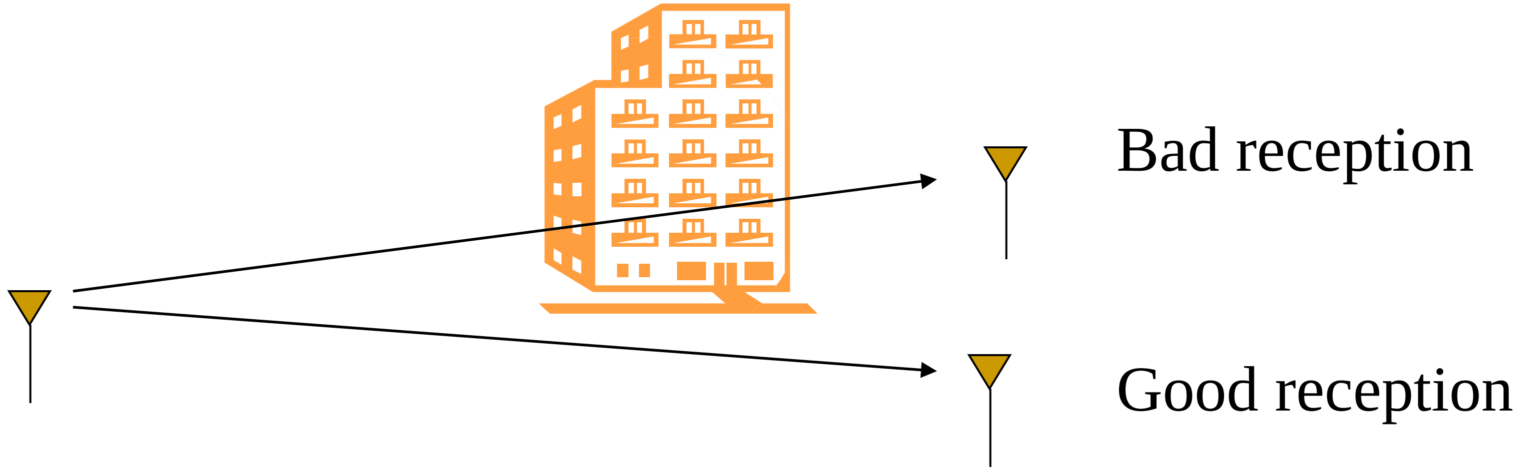
Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

Table 2.2: Typical Path Loss Exponents

Shadowing

- Obstacles between transmitter and receiver
- Signal attenuated





- Random variation of received power due to blockage from objects in signal path
- The exact locations and impact of the blocking objects are usually unknown => statistical models
- The most common model: log-normal pdf
  - Gain in dB is normal.

$$p(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi_{dB}}\psi} \exp\left[-\frac{(10\log_{10}\psi - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}}\right], \ \psi > 0.$$

$$\xi = 10/\ln 10.$$

- $\psi < 1 \Rightarrow P_t < P_r$ physically impossible

$$\mu_\psi = E[\psi] = \exp\left[\frac{\mu_{\psi_{dB}}}{\xi} + \frac{\sigma^2_{\psi_{dB}}}{2\xi^2}\right]$$

- Log-normal model captures the underlying physical model most correctly when $\mu_{\psi_{dB}} >> 0$

- Mathematical justification for log-normal
  - Attenuation due to an object $e^{-\alpha_i}$
  - Attenuation due to many objects $e^{-\sum_i \alpha_i}$
  - CLT after taking logarithm
- Shadowing is a random process
- Assumption: WSS
- Covariance between shadow fading at two points separated by distance δ

- Decorrelation distance X_c is where correlation drops to 1/e of max
- Shadowing r.p.
  - White noise passed through a first-order IIR filter (AR)

Combined Path Loss and Shadowing

- Combination of
  - simplified path loss $\mu_{\psi_{dB}}$
  - Zero mean shadow fading creating variations in received power

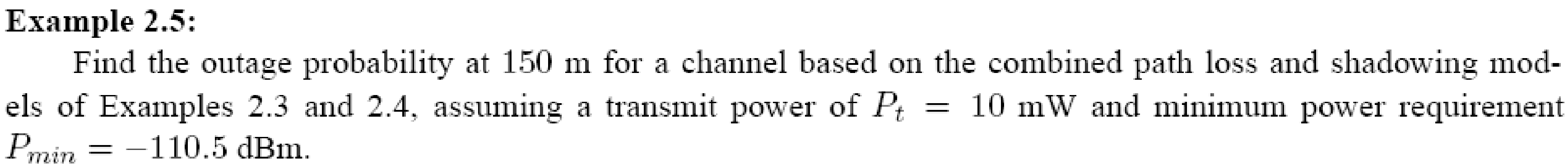
$$\left(\frac{P_r}{P_t}\right)_{dB} = 10\log_{10} K - \underbrace{10\gamma\log_{10}\frac{d}{d_0}}_{\text{Slowly changing}} \underbrace{-\psi_{dB},}_{\text{Rapidly changing}} \quad \psi_{dB} \sim N(0,\sigma^2_{\psi_{dB}})$$

- Outage probability under path loss and shadowing
  - Outage: event that the received power falls below a predetermined power level

$$p(P_r(d) \leq P_{min}) = 1 - Q\left(\frac{P_{min} - (P_t + 10\log_{10} K - 10\gamma \log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right)$$

$$Q(z) \triangleq p(x > z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

  - Outage probability idea can be used to find the cell coverage area in cellular networks.



Example 2.5:

Find the outage probability at 150 m for a channel based on the combined path loss and shadowing models of Examples 2.3 and 2.4, assuming a transmit power of $P_t = 10$ mW and minimum power requirement $P_{min} = -110.5$ dBm.

900MHz, $\gamma = 3.71$, K=-31.54dB, varianceof log-normal shadowing 13.29