Medical Ultrasound Fundamentals

EE-415
INTRODUCTION to
MEDICAL IMAGING

METU

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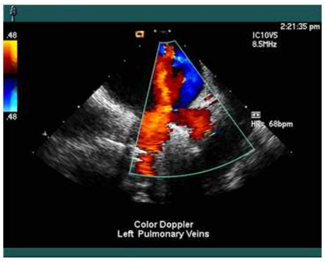

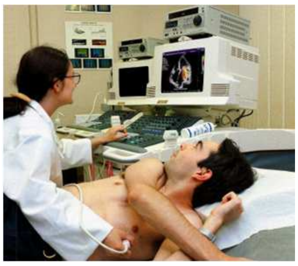

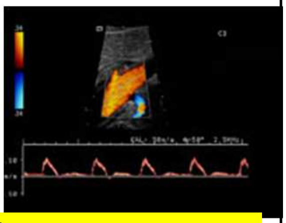

References: Principles of Medical Imaging, by Shung, Smith and Tsui
Foundations of Medical Imaging, by Cho, Jones and Singh
Medical Imaging Physics, by Hendee and Ritenour

History:

- In 1880, Pierre and Jacques Curie discovered the piezoelectric effect.

  "Piezo" is "pressure" in Greek.

  Piezoelectricity refers to the generation of an electrical response to an applied pressure.

- French physicist Paul Langevin attempted to develop piezoelectric materials as senders and receivers of high frequency mechanical disturbances (ultrasound waves) through materials (1915).
  - His specific application was the use of ultrasound to detect submarines during Word War I.
  - This technique, SOund Navigation And Ranging (SONAR), finally become practical during World War II.

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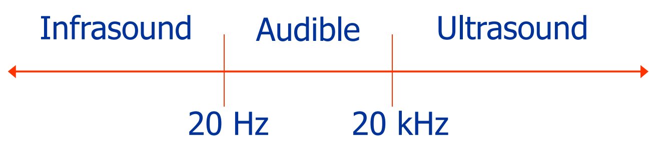
- Industrial uses of ultrasound began in 1928 with the suggestion of Sokolov that it could be used to detect hidden flaws in materials.
- Medical uses of ultrasound through the 1930's were confined to *therapeutic* applications such as cancer treatment and physcial therapy for various illnesses.
- Diagnostic applications of ultrasound began in the late 1940's through collaboration between the physicians and engineers with SONAR.

Acoustic Wave Spectrum



Infra = below, beneath

ultra = beyond, above

Different Forms of Energy

- Electromagnetic
  - Photons, electromagnetic waves
  - Does not require a material medium to propagate
    - Mechanisms of propagation through material media are different from that of propagation through free space
- Acoustic
  - Requires a material medium to propagate
  - *Consists of* oscillatory motions of the atoms/molecules of a material
  - Oscillating particles have kinetic energy proportional to the square of the amplitudes of their motions
  - Through action of intermolecular forces, particles transfer their energy to adjacent particles, yielding energy wave traveling through material.

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Transfer/Transformation of Energy

- Light becomes sound — *photoacoustic* phenomena
- Sound becomes light — *sonoluminescence*
- Absorbed electromagnetic (EM) and acoustic energy both become heat
- Nevertheless, EM and acoustic energy are fundamentally distinct phenomena

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Ultrasound Intensity

- As an ultrasound wave passes through a medium, it transports *energy* through the medium.
- The rate of energy transport is known as ***power**.*
- Medical ultrasound is produced in beams that are usually focused into a small area, and the beam is described in terms of:
  - the power per unit area, defined as the beam's ***intensity*** (Watts/cm^2).
- No universal standard reference intensity exist for ultrasound:
  - "ultrasound at 50 dB was used" is nonsensical.
  - "the returning echo was 50 dB below the transmitted signal" is informative.

The power consumed by a force F that has moved an object by a distance l in time t is given by

$$P = F \cdot \frac{l}{t}$$

Ultrasound is a pressure wave.

Power P carried by an ultrasonic wave is

$$P = \underbrace{F}_{\text{force exerted by the pressure wave}} \times particle\ velocity$$

Thus the instantaneous intensity (power carried by the wave per unit area) can be expressed as

$$i(t) = \underbrace{\frac{Force}{Area}}_{\text{pressure}} \cdot particle\ \ velocity = \underbrace{p(t)}_{\text{pressure}} \cdot u(t)$$

Average intensity (Sinusoidal propagation) :

Let: $p(t) = P_m \text{Sin}\,\omega t$ and $u(t) = U_m \text{Sin}\,\omega t$

$$I = \frac{1}{T}\int_0^T P_m Sin\,\omega t\, U_m Sin\,\omega t\, dt$$

$$= P_m U_m \frac{1}{T}\int_0^T Sin^2\omega t\, dt = P_m U_m \frac{1}{T}\cdot\frac{T}{2}$$

$$= \frac{1}{2} P_m U_m$$

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Average intensity (Sinusoidal propagation) :

Electrical Impedance:

$$\boxed{Z_{el} = V_m/I_m}$$

Characteristic acoustic impedance of the medium:

$$\boxed{Z = P_m/U_m = \rho c}$$

ρ: mass density of the medium

c: velocity of ultrasound

$$\boxed{I = \frac{1}{2} P_m U_m = \frac{1}{2}(\rho c) U_m^2 = \frac{1}{2}\frac{P_m^2}{(\rho c)}}$$

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Safety limits

Maximum ultrasound intensities recommended by the U.S. Food and Drug Administration (FDA) for various diagnostic applications.

Application	Max. Intensity (mW/cm^2)
Cardiac	430
Peripheral vessels	720
Opthalmic	17
Abdominal	94
Fetal	94

Ultrasound velocity

- The velocity of ultrasound wave through a medium varies with the physical properties of the medium.
  - **Low-density media** (air and other gases): molecules may move over relatively large distances before they influence neighboring molecules.

    $\Rightarrow$ **the velocity** of ultrasound wave is **low**.
  - **High-density media** (solids): molecules are constrained in their motion.

    $\Rightarrow$ **the velocity** of ultrasound wave is **high**.
  - **Liquids** exhibit ultrasound velocities intermediate between those in gases and solids.
- In different media, changes in velocity are reflected in changes in wavelength of the ultrasound waves, with the frequency remaining relatively constant.

Attenuation of Ultrasound

As an ultrasound beam penetrates a medium, energy is removed from the beam by

- absorption,
- scattering, and
- reflection

As with x-rays, the term "**attenuation**" refers to any mechanism that removes energy from the ultrasound beam.

Ultrasound is "**absorbed**" by the medium if part of the beam's energy is converted into other forms of energy, such as an increase in the random motion of molecules.

If the **obstacle's size is large** compared to the wavelength of sound then part of the beam may be "**reflected**" and the remainder "**transmitted**" through the obstacle as a beam of lower intensity.

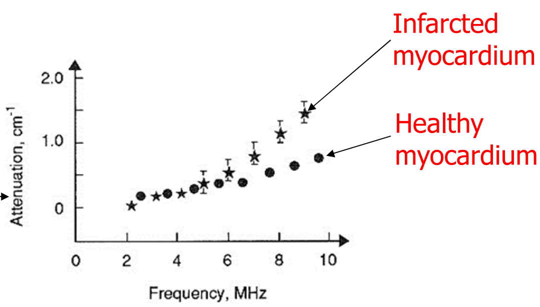
If the size of the obstacle is **comparable to or smaller than** the wavelength of the ultrasound, the obstacle will "**scatter**" energy in various directions.

Attenuation coefficients α for 1 MHz Ultrasound

Material	α (dB/cm)	Material	α (db/cm)
Blood	0.18	Lung	40
Fat	0.6	liver	0.9
Muscle (across fibers)	3.3	Brain	0.85
Muscle (along fibers)	1.2	Kidney	1
Aqueous and vitreous humor of eye	0.1	Spinal cord	1
Lens of eye	2.0	water	0.0022
Skull bone	20	Caster oil	2

Clinical Potential of Attenuation Measurements

Note, overall attenuation coefficient β, not only absorption or only (back)scattering



Ultrasound attenuation and backscatter measurements can be used (among many other things) to assess extent of tissue death in myocardial infarction

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Reflection

- In most diagnostic applications of ultrasound, ultrasound waves **reflected from interfaces** between different tissues in the patient is used.
- The fraction of the energy reflected from an interface depends on the difference in acoustic impedance of the media on opposite sides of the interface.
- The acoustic impedance Z of a medium is the product of the **mass density** of the medium and the **velocity of ultrasound** in the medium:

$$Z = \rho c$$

An alternative definition:

Acoustic impedance = pressure / particle velocity

Electrical circuit analogue :

impedance = voltage / current

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Velocity and acoustic impedance of pertinent materials and biological tissues at room temperature (20–25°C)

	Velocity (m/sec)	Impedance $\times 10^{-6}$ (kg/m²-sec)[a]
Water	1484	1.48
Aluminum	6420	17.00
Air	343	0.0004
Plexiglas	2670	3.20
Blood	1550	1.61
Myocardium (perpendicular to fibers)	1550	1.62
Fat	1450	1.38
Liver	1570	1.65
Kidney	1560	1.62
Skull bone	3360 (longitudinal)	6.00

[a] Rayl is a unit commonly used for acoustic impedance. One rayl = 1 kg/m²-sec.

metal

gas

acrylic

soft tissues

hard tissue

Notice how similar these values are to each other and to that for water

$$\boxed{\text{rayl} = \rho c = (\text{kg/m}^3)(\text{m/sec}) = \text{kg-m}^{-2}\text{-sec}^{-1}}$$

Refraction

- As an ultrasound beam crosses an interface obliquely (not orthogonal) between two media, its direction is changed (i.e., the beam is bent).
  This behavior of ultrasound transmitted obliquely across an interface is termed *refraction*.

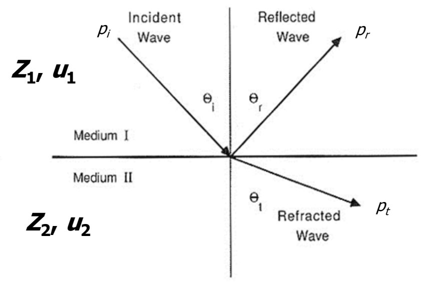
- The relationship between the incident and refraction angles is decribed by the **Snell's law**:

$$\frac{Sin\,\theta_i}{Sin\,\theta_t} = \frac{u_i}{u_t}$$

- The incidence angle at which refraction causes no ultrasound to enter a medium is termed the critical angle: $\theta_{i.} = \theta_c$ (consider $\theta_{t.} = 90^o$, sin $\theta_{t.} = 1$)

Reflection and Refraction

- Behavior of ultrasound at an interface between materials of **different *Z*** is analogous to behavior of light at interface between materials of different refractive index.
- The Snell law applies (as in optics) if the wavelength of the wave is much smaller than the dimensions of the interface
- Fraction of pressure reflected = Reflection Coefficient, *R*
- Fraction of pressure transmitted = Transmission Coefficient, *T*
- $\theta_i = \theta_r$ and $\sin\theta_i / \sin\theta_t = u_1 / u_2$



$$p_r = R\,p_i$$

$$p_t = T\,p_i$$

Figure 69 Reflection and refraction of plane wave at a flat boundary.

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- In a propagating wave, there are no sudden discontinuities in either particle velocity (u) or particle pressure (p). Consequently, when a wave meets the interface between two media, both the particle velocity and the pressure are **continuous** across the interface. These conditions are satisfied when

$$u_i \cos\theta_i - u_r \cos\theta_r = u_t \cos\theta_t$$

Use the perpendicular component for u

and

$$p_i + p_r = p_t$$

See Azhami, (especially Sec. 6.3, p. 115)

Since p=Zu, and $\theta_i = \theta_r$, it is possible to obtain the following relations:

$$R = \frac{p_r}{p_i} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t},$$

$$T = \frac{p_t}{p_i} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t}.$$

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- *Intensity* reflection and transmission coefficients are derived from the preceding equations and using the relations
  $p = Zu$ and $I = p^2/(2Z)$.

$$\frac{I_r}{I_i} = \left(\frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t} \right)^2 , \quad \frac{I_t}{I_i} = \frac{4 Z_1 Z_2 \cos^2\theta_i}{\left(Z_2 \cos\theta_i + Z_1 \cos\theta_t\right)^2} .$$

Normally incident wave (i.e., $\theta_i = \theta_t = 0$) :

$R= (Z_2-Z_1)/(Z_2+Z_1)$
$T=2Z_2/(Z_2+Z_1)$

$I_r/I_i = (p_i^2/2Z_1)/(p_r^2/2Z_1)$
$I_t/I_i = (p_t^2/2Z_2)/(p_i^2/2Z_1) = 4Z_1Z_2/(Z_2+Z_1)$

- If $Z_1=Z_2$, $p_r/p_i=0$, and there is no reflected wave,
- If $\mathbf{Z_2>Z_1}$, the **reflected** pressure wave is **in phase** with the incident wave,
- If $\mathbf{Z_2<Z_1}$, the **reflected** wave is **180 degrees out of phase** with the incident wave.

Piezoelectric Effect

- The piezoelectric effect is exhibited by certain crystals that, in response to applied pressure, develop a voltage across opposite surfaces. This effect is used to produce an electrical signal in response to incident ultrasound waves.
- Similarly, application of voltage across the crystal causes deformation of the crystal. This deforming effect, termed the converse piezoelectric effect, is used to produce an ultrasound beam from a transducer.
- Many crystals exhibit the piezoelectric effect at low temperatures, but are not suitable as ultrasound transducers because their piezoelectric properties do not exist at room temperature.
- The **temperature** above which a crystals's piezoelectric properties disappear is known as Curie point of the crystal.

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Piezoelectric Properties

- Efficiency of the transducer is the fraction of applied energy that is converted to the desired energy mode (a measure of the ability of a transducer to convert one form of energy from another).
- For an ultrasound transducer, this definition of efficiency is described as the **electromechanical coupling coefficient** k_c .
- If mechanical energy (i.e., pressure) is applied, we obtain

$$k_c^2 = \frac{\text{mechanical energy converted to electrical energy}}{\text{applied mechanical energy}}$$

- If electrical energy is applied, we obtain

$$k_c^2 = \frac{\text{electrical energy converted to mechanical energy}}{\text{applied electrical energy}}$$

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Properties of selected piezoelectric crystals

Materials	Electromechanical coupling coefficient (k_c)	Curie point (°C)
Quartz (occur in nature)	0.11	550
Rochelle salt (occur in nature)	0.78	45
Barium titanate (man-made)	0.30	120
Lead zirconate titanate (PZT-4) (man-made)	0.70	328
Lead zirconate titanate (PZT-5) (man-made)	0.70	365

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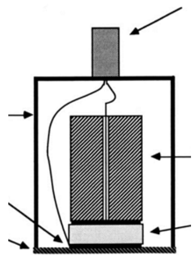
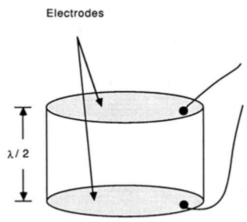
Transducer design

- The piezoelectric crystal is the functional component of an ultasound transducer.
- A crystal exhibits its greatest response at the resonance frequency.
- The resonance frequency is determined by the thickness t of the crystal (the dimension of the crystal along the axis of the ultrasound beam).
- A crystal of half-wavelength thickness resonates at a frequency ν:



Ultrasound wavelength in disk material

$$\nu = \frac{c}{\lambda} = \frac{c}{2t}$$



Piezoelectric disc coated with silver electrodes

Example:
a 1.5 mm thick quartz disk
(c =5740 m/sec in quartz acoustic wave velocity)

has a resonance frequency of

ν = 5740/(2 x 0.0015)
= 1.91 MHz.

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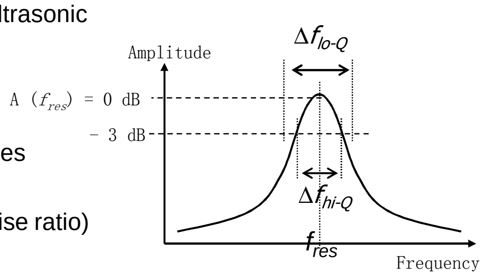
Transducer Q-factor

- Disc of piezoelectric material (usually PZT) resonates at mechanical resonance frequency f_{res}

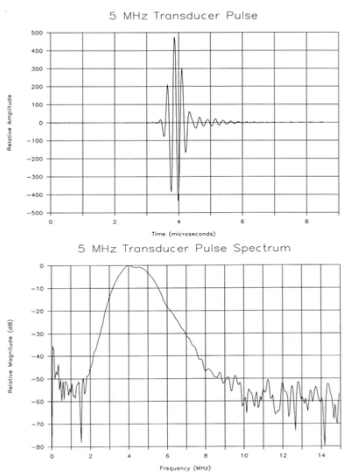
$\rightarrow$ Resonance curve (Q-factor, $Q = f_{res}/\Delta f$; Δf is -3 dB width of curve)

- High Q: strong resonance (narrow BW)
  Desirable for continuous wave ultrasonic Doppler flowmeter
- Low Q: strongly damped,
  weak resonance (broad BW)
  Desirable for pulse-echo devices
- Tradeoff of high Q:
  - + Efficient at f_{res} (high signal-to-noise ratio)
  - – Pulse distortion (ringing effect)



Transducers in Pulsed / C.W. Mode

- Low bandwidth:
  - No backing
  - High efficiency
  - High-Q
  - Strong “Pulse ringing”
  - Used for C.W. applications
- Large Bandwidth:
  - Backing
  - Low-Q
  - Lowered efficiency
  - Used for pulsed applications

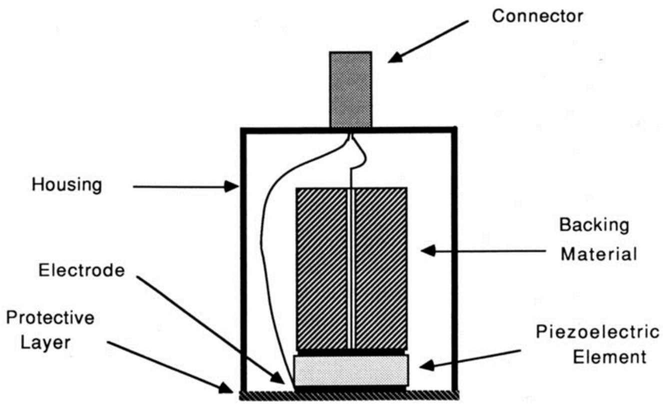


The characteristics of a 5MHz transducer for pulsed applications (Low Q)

Top: time response,

Bottom: frequency response

Typical Ultrasound Transducer



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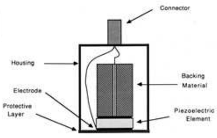
Transducer Backing



- With only air behind the crystal, ultrasound transmitted back into the cylinder from the crystal is reflected from the cylinder's opposite end.
- The reflected ultrasound reinforces the ultrasound propagated in the forward direction.
- This reverberation of ultrasound in the transducer itself contributes energy to the ultrasound beam (i.e., it increases the efficiency).
- It also extends the time over which the ultrasound pulse is produced.
- Extension of the pulse duration (decreases bandwidth, increases Q) is not a problem in some clinical applications such as continuous wave applications.
- However, most ultrasound imaging applications utilize short pulses of ultrasound, and suppression of ultrasound reverberation is desirable.
- Backing of transducer with an absorbing material (tungsten powder embedded in epoxy resin) reduces reflections from back, causes damping at resonance frequency
  - Reduces the efficiency of the transducer
  - Increases Bandwidth (lowers Q)

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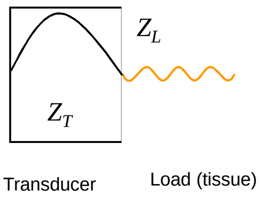
Transducer – Tissue Mismatch (See Sec. 6.3.3 from Azhami)

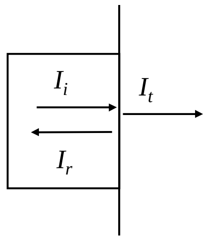
- Impedance mismatch causes reflection, inefficient coupling of acoustical energy from transducer into tissue:

  $Z_T \approx 30$ M rayl
  $Z_L \approx 1.5$ M rayl $\Rightarrow I_t/I_i = 0.18$

$$\frac{I_t}{I_i} = \frac{4Z_T Z_l}{\left(Z_T + Z_l\right)^2}$$

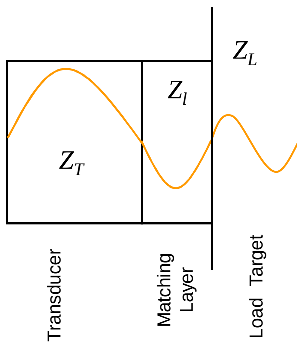
- Solution: Matching layer(s)
  - increases coupling efficiency
  - damps crystal oscillations, increases bandwidth (reduces efficiency)





Matching Layers

- A layer between transducer and tissue with $Z_T > Z_l > Z_L$ creates stepwise transition
- *Ideally*, 100 % coupling efficiency across a matching layer is possible if
  - layer thickness = $\lambda/4$
  - and $Z_l = \sqrt{Z_T Z_L}$
- Problems: Finding material with exact Z_l value (~6.7 MRayl)



Axial beam profile

- **Ultrasound sources** may be considered to be a **collection of point sources**, each radiating **spherical wavelets** into the medium.
- **Interference of the spherical wavelets** establishes a characteristic pattern for the resulting **wavefronts**.
- The reinforcement and cancellation of individual wavelets are most noticable in the region near the source of ultrasound.
- They are progressively less dramatic with increasing distance from the ultrasound source.

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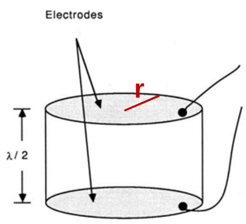
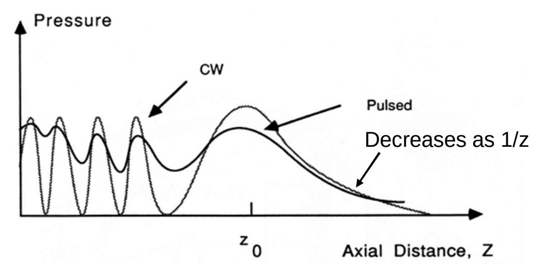
Axial beam profile

- The region **near the source** where the interference of wavelets is most apparent is termed the **Fresnel (or near) zone**.
- For a disk-shaped transducer of radius r, the length Z_0 (the distance between the transducer and the last maximum of the axial pressure) of the Fresnel zone is:

$$Z_0 = \frac{r^2}{\lambda}$$





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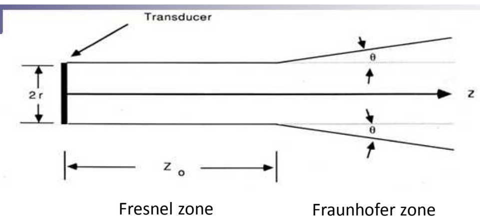

- Within the **Fresnel zone**, most of the ultrasound energy is confined to a beam width **no greater than the transducer diameter**.

- Beyond the Fresnel zone, some of the energy escapes along the preriphery of the beam to produce a gradual **divergence of the ultrasound beam** that is described by

$$\sin\theta = 0.6\frac{\lambda}{r}$$

where θ is the **Fraunhofer divergence angle** in degrees.

- The region beyond the Fresnel zone is termed the **Fraunhofer (or far-field) zone**.

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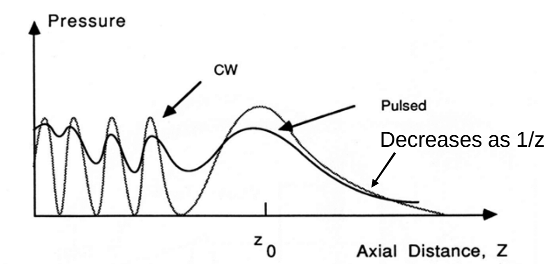
Axial beam profile

- In the **near-field (Fresnel) zone**, axial pressure oscillates.

- In the **far-field (Fraunhofer) zone**, axial pressure decreases approximately according to 1/z.



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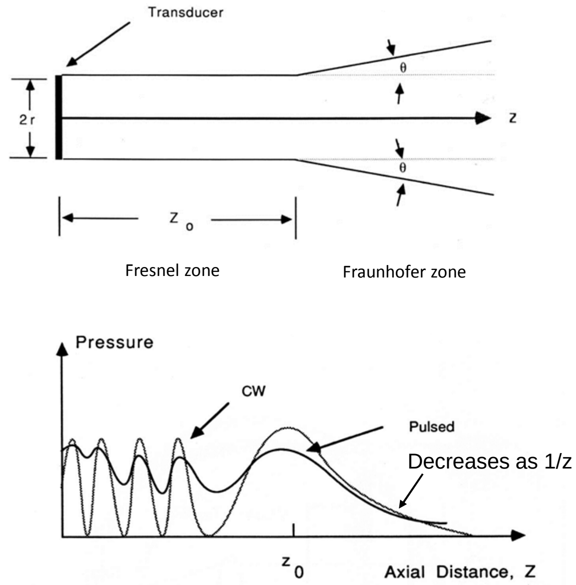
Axial beam profile



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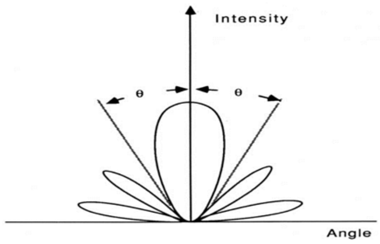
Angular radiation pattern of intensity in the far-fied

- Isoecho contours: each contour depicts the locations of equal echo intensity for the ultrasound beam.
- At each of these locations, a reflecting object (small steel ball) will be detected with equal sensitivity.
- Connecting these locations with lines yields isoecho contours.

$$\sin\theta = 0.61\frac{\lambda}{r}$$



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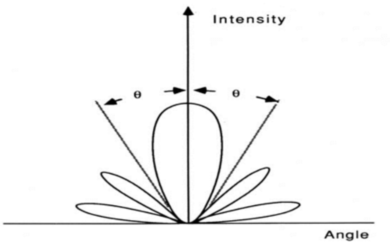
Angular radiation pattern of intensity in the far-fied

- Angular radiation pattern of intensity in the far field of an US transducer consists of a main lobe and several side lobes.
- The first zero – the angle at which the main lobe becomes zero: θ
- The number of side lobes and their magnitude relative to that of the main lobe depends on λ / r.
- Side lobes are very undesirable; spurious signals resulting in image artifacts

$$\sin\theta = 0.61\frac{\lambda}{r}$$



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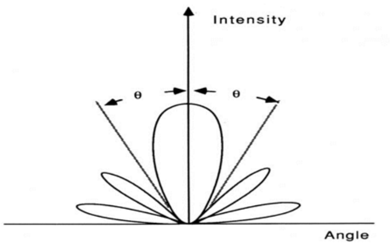
Angular radiation pattern of intensity in the far-fied

- Small λ/r:
  - Small θ
  - Large Z_o
  - Sharper main lobe
  - Better image resolution

  - More side lobes
  - More artifacts due to side lobes

  - COMPROMISE!!



$$\sin\theta = 0.61\frac{\lambda}{r}$$

$$Z_0 = \frac{r^2}{\lambda}$$

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Rules for Transducer design

v = c/λ, λ = c/v

$$\boxed{Z_0 = \frac{r^2}{\lambda} = \frac{r^2 \upsilon}{c}}$$

$$\boxed{\sin\theta = 0.6\frac{\lambda}{r} = 0.6\frac{c}{r\,\upsilon}}$$

- For a given transducer diameter,
  - the near field length (Z_0) increases with increasing frequency
  - beam divergence (sin θ) in the far field decreases with increasing frequency,
- For a given transducer frequency,
  - the near field length increases with increasing transducer diameter,
  - beam divergence in the far field decreases with increasing transducer diameter.

Rules for Transducer design

Example: What is the length of the Fresnel zone for a 10-mm diameter, 2MHz unfocused ultrasound transducer?

λ = (1540 m/sec) / (2x10[6]/sec) = 0.77 mm.

Z_0 = (5mm)^2 / 0.77 mm = 32.5 mm

v = c/λ, λ = c/v

$$Z_0 = \frac{r^2}{\lambda} = \frac{r^2 \upsilon}{c}$$

$$\sin\theta = 0.6\frac{\lambda}{r} = 0.6\frac{c}{r\,\upsilon}$$

Transducer radius and ultrasound frequency and their relationship to Fresnel zone and beam divergence

Frequency (Mhz)	Wavelength (cm)	Fresnel zone (cm)	Fraunhofer divergence angle (degrees)
Transducer radius constant at 0.5 cm			
0.5	0.30	0.82	21.5
1.0	0.15	1.63	10.5
2.0	0.075	3.25	5.2
4.0	0.0325	6.5	2.3
8.0	0.0163	13.0	1.1
Radius(cm)		Fresnel zone (cm)	Fraunhofer divergence angle (degrees)
Frequency constant at 2 MHz			
0.25	0.075	0.83	10.6
0.5	0.075	3.33	5.3
1.0	0.075	13.33	2.6
2.0	0.075	53.33	1.3

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- **Note that** near field length increases with frequency but **absorption also rises with frequency**.
- There will be a maximum depth for detecting echos with ultrasound of particular frequency.
- Therefore, with increasing frequency resolution improves but penetration depth decreases as a result of increased attenuation.

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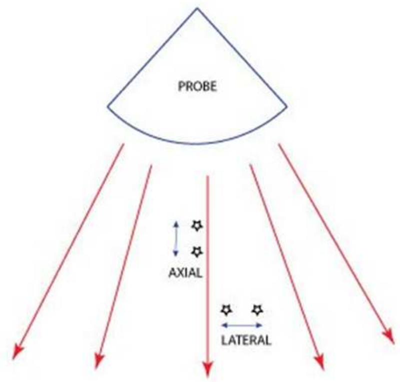
Axial and Lateral Resolution



http://echocardiographer.org/Echo Physics/Axresolution.html

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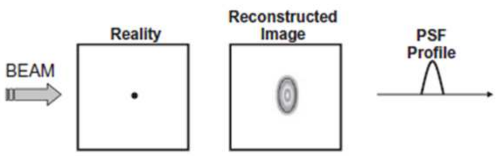
Axial and Lateral Resolution



- An ultrasonic beam, which is schematically represented as an arrow, scans a point target.
- The obtained image is the corresponding PSF of the system, which is schematically represented as an ellipse.
- This ellipse – type image stems from the fact that the smearing along the lateral direction is typically much greater than along the axial direction.
- The profile of the PSF is schematically represented in the right - hand column.

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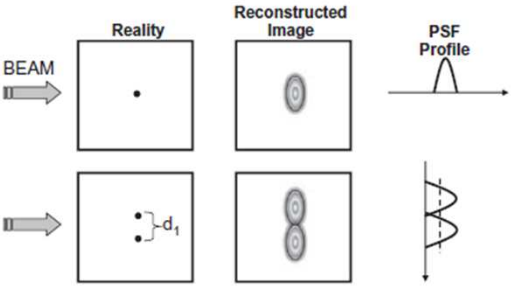
Lateral Resolution



- In order to estimate the lateral resolution of a system, two point targets separated by distance, d_1 along the lateral direction are scanned.
- If the resolution of the system is sufficient, the reconstructed image will depict two adjacent spots.
- By plotting the central profiles along the lateral direction of these spots, two adjacent (or even overlapping) profiles will be obtained.
- The two spots in the image may be considered separated if cutting these profiles at 50% of their maximal amplitude, the profiles do not overlap.

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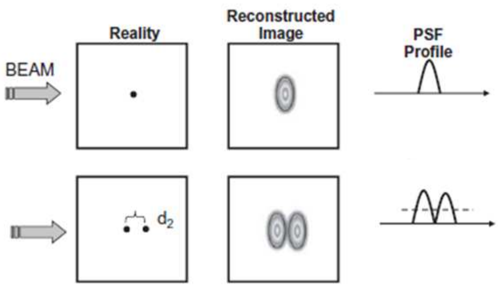
Axial Resolution



- Similarly, the resolution along the axial direction d_2 can be estimated by scanning two point targets positioned one after the other.

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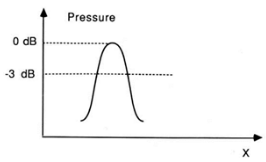
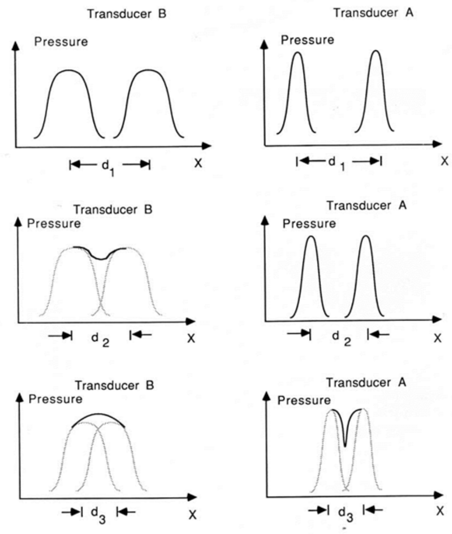
Relationship with beam width





Transducer A has a narrower beam width, thus a better resolution than transducer B.

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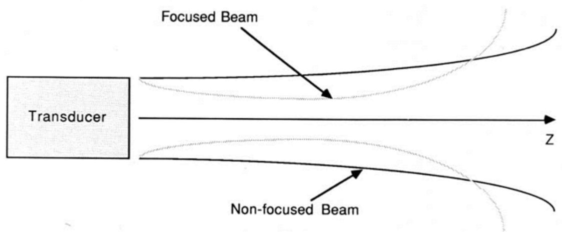
Focusing of Ultrasound

- Improve lateral resolution by reducing the beam width
- Increased spatial resolution at specific depth
- However, an improvement in the lateral resolution in a certain range always accompanies a loss of resolution in other regions.
  - Focusing of trancducer reduces beamwidth in limited region of beam but results in more rapid divergence beyond focal zone.



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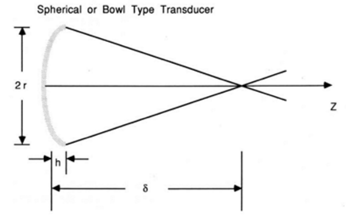
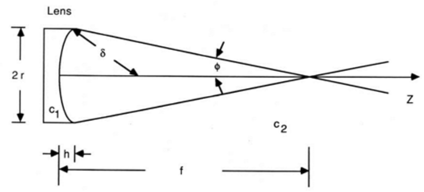
Focusing of Ultrasound

- Self-focusing transducer:



- Focusing with aid of lens:



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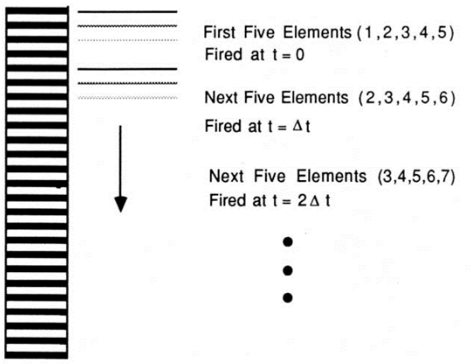
Transducer Arrays

- Switched Array: Linear switched array produces images by successively exciting groups of piezoelectric elements.
- In this way, the sound beam is moved accross the face of the transducer electronically, producing a picture similar to that obtained by scanning a single transducer manually.



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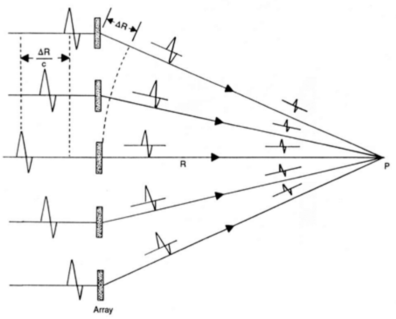
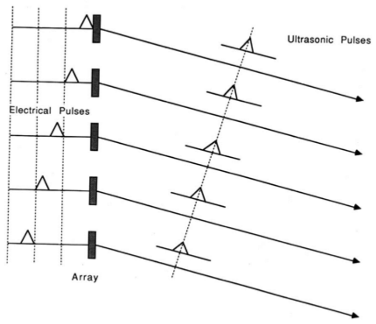
Transducer Arrays

- Phased Array for beam steering, focusing

Linear phased array focuses beam in transmission by appropriately delaying excitation pulses to different elements.



Steering of beam produced by linear phased array.



By appropriately adjusting the delays, beam steering and focusing can be produced simultaneously.

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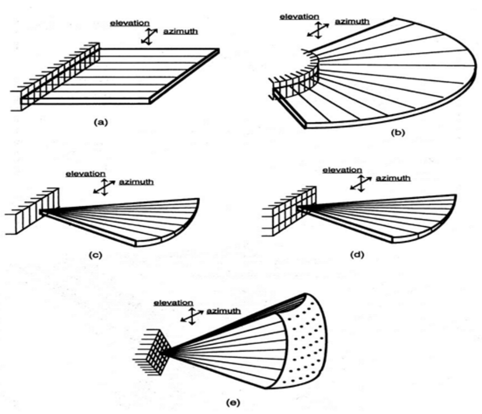
Array Types

a) Linear Sequential (switched) ~1 cm × 10-15 cm, up to 512 elements
b) Curvilinear similar to (a), wider field of view
c) Linear Phased up to 128 elements → cardiac imaging
d) 1.5D Array 3-9 elements in elevation allow for focusing
e) 2D Phased Focusing, steering in both dimensions



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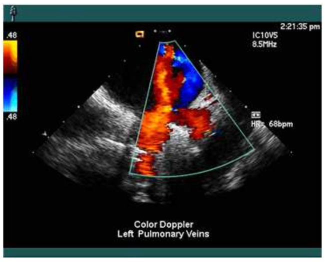
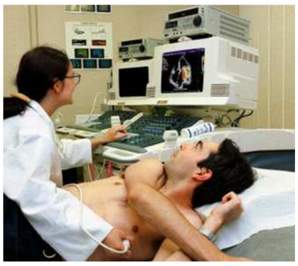
Ultrasound Imaging

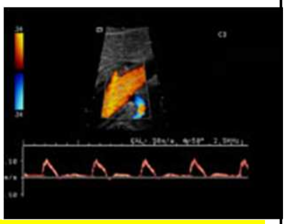
EE-415
INTRODUCTION to
MEDICAL IMAGING

METU

Instructor:
Yeşim Serinağaoğlu







References: Principles of Medical Imaging, by Shung, Smith and Tsui
Foundations of Medical Imaging, by Cho, Jones and Singh
Computerized Tomography, by Slaney and Kak

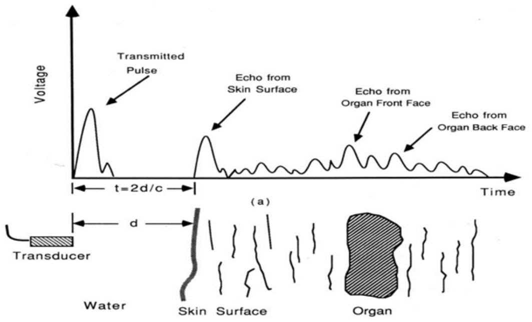
Pulse-Echo Systems

- A single probe is used both for transmitting and receiving the echoes reflected (or scattered back) from the tissues.
- To obtain the best axial resolution, the probe is excited by extremely short pulses.
- Depending on how the information displayed, pulse-echo methods can be classified as:
  - A-mode
  - B-mode
  - M-mode

See: http://www.ultrasonix.com/wikisonix/index.php/Ultrasound_Image_Computation

A Mode (Amplitude Mode)

- Oldest, simplest type
- Display of the envelope of pulse-echoes vs. time, depth $d = c(t/2)$
- Pulse repetition rate ~ kHz
  (which is limited by penetration depth,
  $c \approx 1.5$ mm/µsec $\Rightarrow$ 20 cm $\approx$ 270 µsec,
  plus an additional wait time $\approx$ 1 msec)



A-mode display

Organ situated beneath skin surface

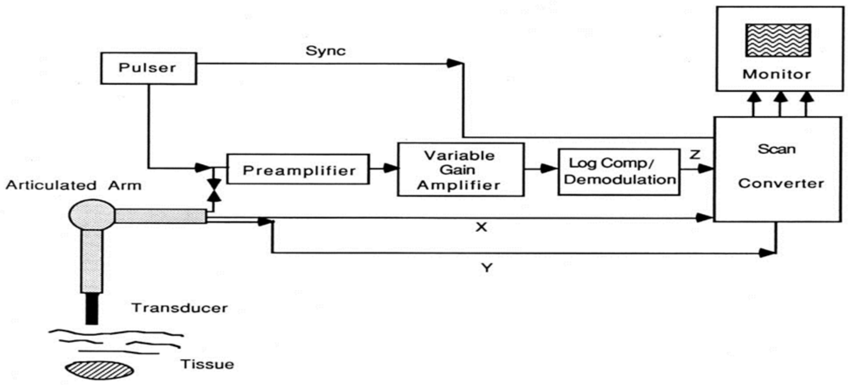
B Mode ("Brightness Mode")

- The location of echo-producing interfaces is displayed in two-dimensions (x,y) on a video screen.
- The amplitude of each echo is represented by the brightness value at the (x,y) location.
- Lateral scan across tissue surface is obtained by means of recording a sequential series of elementary image lines.
- The time required to obtain a single image line extending to depth d in the object is $t = 2d / c$.
- The minimum time reqired to obtain an N-line image is, $t = 2Nd / c$.

where **c** is the velocity of sound in the material.

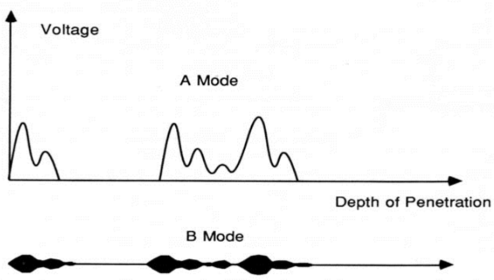
B Mode ("Brightness Mode")

- The block diagram of a static B-mode scanner

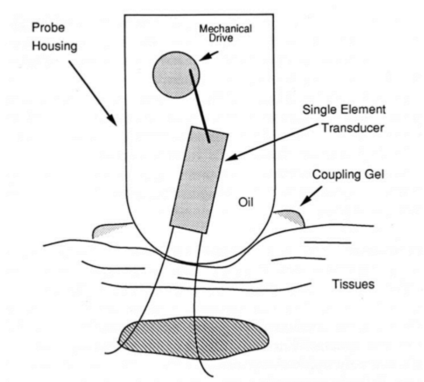


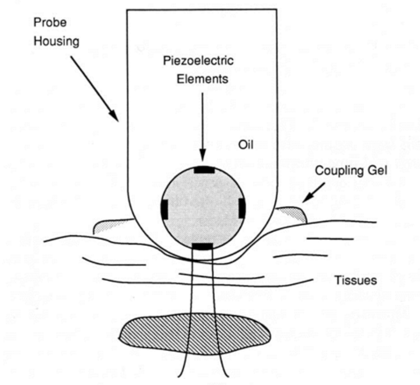
B Mode ("Brightness Mode")

- Traditional B-Mode images are 2D cross-sectional images of tissue acquired using 1D transducer arrays.
- A cross sectional image of the tissue can be formed by placing the A-Mode data for successive scan-lines side by side to form a 2D array of data.



Real-Time B Scanners





- Frame rate R_f ~30 Hz

  $R_f = c / (2d \times N)$ d: depth, N: no. of lines

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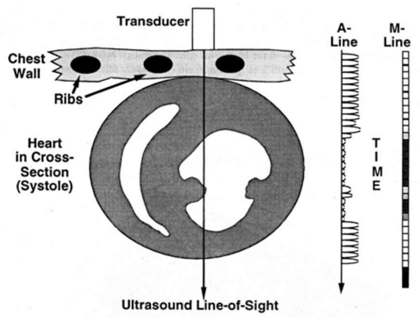
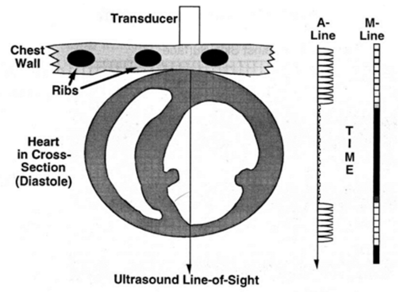
M-Mode ("Motion Mode")

- M-Mode images are images that show the motion of different points of tissue along a SINGLE scan line as a function of time.
- To generate the M-Mode images, the A-mode data from successive acquisitions (frames) of the same scan line are placed side by side in an array as an image.
- The image is updated in real-time as newer data become available.
- (cardiac imaging: wall thickness, valve function)





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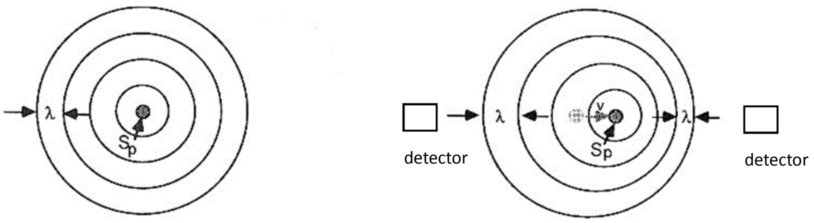
Doppler Effect

- When there is relative motion between a source and a detector of ultrasound, the frequency of the detected ultrasound differes from the frequency of the ultrasound emitted by the source.



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Doppler Effect

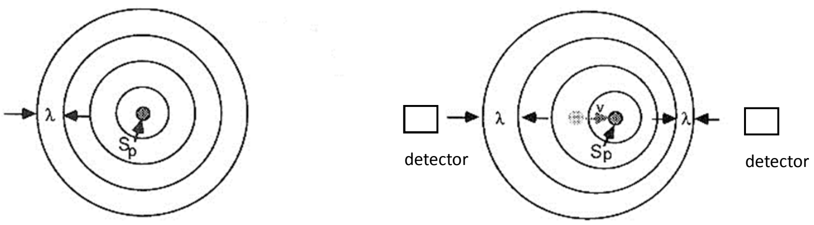
- An ultrasound source is moving with velocity v_s **towards** the detector.

- After time *t*, following the production of any wavefront,
  - the **distance between** the wavefront and the source is

    $(c\text{-}v_s)t$,

    where *c* is the velocity of the ultrasound in the medium.

- The **wavelength** λ_m of the ultrasound in the direction of motion **is shortened** $\lambda_m=(c-v_s)/f_0$ where f_0 is the frequency of the ultrasound from the source.



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- With the shortened wavelength, the ultrasound reaches the detector with an increased frequency *:*

$$f = \frac{c}{\lambda_m} = \frac{c}{(c - v_s)/f_0}$$
$$= f_0\left(\frac{c}{c - v_s}\right)$$

- That is, the frequency of the detected ultrasound shifts to a higher value when the ultrasound source is moving toward the detector.
- The shift in the frequency is:

$$\Delta f = f - f_0 = f_0\left(\frac{c}{c - v_s}\right) - f_0$$
$$= f_0\left(\frac{v_s}{c - v_s}\right)$$

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$$\Delta f = f_0\left(\frac{v_s}{c - v_s}\right)$$

- If the velocity *c* of ultrasound in the medium is much greater than the velocity v_s of the ultrasound source, then c-v_s~ *c* and

$$\Delta f = f_0\left(\frac{v_s}{c}\right)$$

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- A similar expression is applicable to the case in which the ultrasound source is stationary and the detector is moving toward the source with velocity v_d.

$$\Delta f = f_0\left(\frac{v_d}{c - v_d}\right)$$

- For $c \gg v_d$, the Doppler shift frequency is approximately

$$\Delta f = f_0\left(\frac{v_d}{c}\right)$$

HW: show!

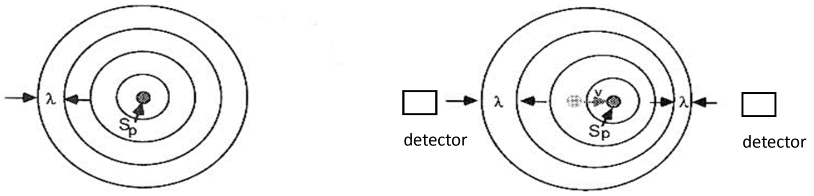
- If the ultrasound source is **moving away** from the detector, then the distance between the source and a wavefront is

  $ct + v_s t = (c + v_s)t$,

  where t is the time elapsed since the production of the wavefront.

- The wavelength λ_m of the ultrasound is $\lambda_m = (c+v_s)/f_0$ and the apparent frequency f is:

$$f = \frac{c}{\lambda_m} = \frac{c}{(c+v_s)/f_0} = f_0\left(\frac{c}{c+v_s}\right)$$



- That is, the frequency shifts to a lower value when the ultrasound source is moving away from the detector.

- The shift in frequency is :

$$\Delta f = f - f_0 = f_0\left(\frac{c}{c+v_s}\right) - f_0$$
$$= f_0\left(\frac{-v_s}{c+v_s}\right)$$

- If the velocity c of ultrasound in the medium is much greater than the velocity v_s of the ultrasound source, then $c+v_S \sim c$ and

$$\Delta f = f_0\left(\frac{-v_s}{c}\right)$$

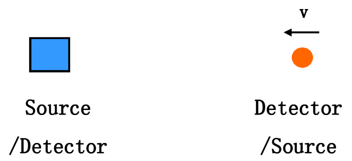
- A similar expression is applicable to the case in which the ultrasound source is stationary and the detector is moving away from the source with velocity v_d. In this case, the Doppler shift frequency is approximately

$$\Delta f = f_0\left(\frac{-v_d}{c}\right)$$

HW: show!

where $c >> v_d$.

- If the source and detector are at the same location, and ultrasound is reflected from an object moving toward the location with velocity v, the object first acts as a moving detector while it receives the ultrasound signal, and then as a moving source as it reflects the signal.



- As a results the ultrasound signal received by the detector exhibits a frequency shift (when c>>v)

$$\Delta f = 2 f_0 \left(\frac{v}{c} \right)$$

HW: show!

- Similarly, for an object moving away from the source and detector, the shift in frequency is

$$\Delta f = 2 f_0 \left(\frac{-v}{c} \right)$$

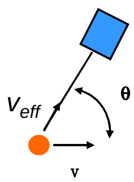
HW: show!

where the negative sign indicates that the frequency of the detected ultrasound is lower than that emitted by the source.

- For the more general case where the ultrasound beam strikes a moving object at an angle θ,

v_{eff}

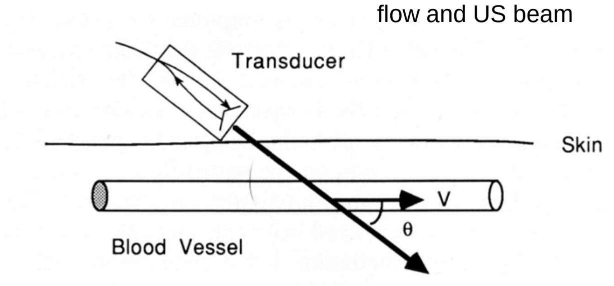
$$\Delta f = 2 f_0 \left(\frac{v}{c} \right) \cos\theta$$



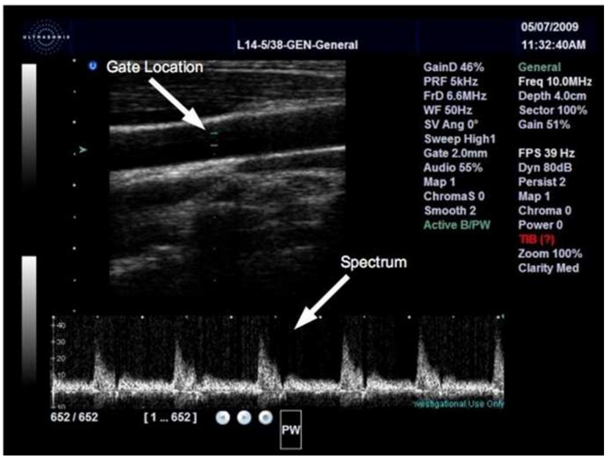
CW Doppler

- Doppler shift in detected frequency

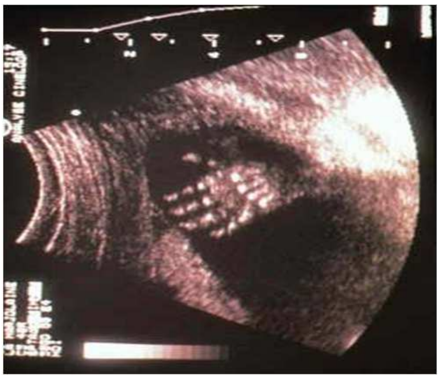
v: blood flow velocity
c: speed of sound
θ: angle between direction of blood flow and US beam



- Pulsed Waved Doppler Imaging, or PW Mode, is a method to use ultrasound for determining blood velocity and direction.



ULTRASONIC COMPUTED TOMOGRAPHY



- Ultrasound CT is very similar to X-ray CT. In both cases, a transmitter illuminates the object and a line integral of the attenuation can be estimated by measuring the energy on the far side of the object.
- Ultrasound differes from X-rays because
  - the propagation speed is much lower;
  - it is possible to measure the exact pressure of the wave as a function of time.
- From the pressure waveform it is **possible to measure**
  - The **attenuation** of the pressure field,
  - The **delay in the signal** induced by the object.
- Thus from these measurements, it is possible to estimate
  - the attenuation coefficient of the object,
  - refractive index of the object
- It is clear that in computerized tomography, it is essential to know the ray path from the source to the detector. In ultrasound, the paths are not always straight lines.

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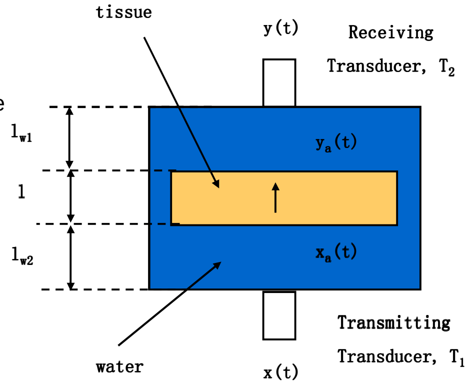
Fundamental considerations

- Ultrasonic waves in the range of 1-10MHz are highly attenuated by air, thus the tissue is immersed in water.
- Water;
  - serves to couple the energy of the transducer into the object,
  - provides a good refractive index match with the tissue.
- If an electrical signal, **x(t)** is applied to the transmitting transducer, a number of effects can be identified that determine the electrical signal produced by the receiving signal.
- We can write an expression for the received signal **y(t),** by considering each of these effects in the **frequency domain.**



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The Fourier Transform of the **received signal Y(f),** is given by a simple multiplication of the following factors:

1) the **transmitter transfer function** relating the electrical signal to the resulting pressure wave, **$T_1(f)$;**

2) the **attenuation $\exp[-\alpha_w(f)l_{w1}]$,** and **phase change $\exp[-j\beta_w(f)l_{w1}]$,** caused by the near side of the tissue,

3) the **transmittence of the front surface** of the tissue or the percentage of energy in the water that is coupled into the tissue, τ_1

4) the **attenuation $\exp[-\alpha(f)l]$,** and **phase change $\exp[-j\beta(f)l]$,** caused by the near side of the tissue,

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5) the **transmittence of the rear surface** of the tissue, τ_2.

6) the **attenuation exp[-α_w(f)l_{w2}],** and **phase change exp[-jβ_w(f)l_{w2}],** caused by the near side of the tissue,

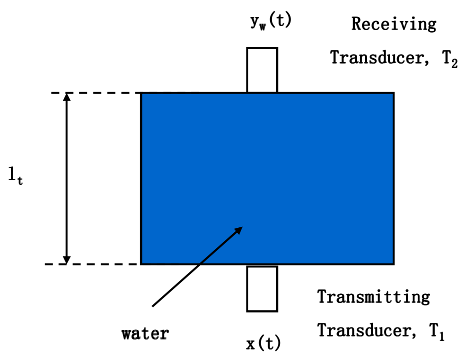
7) the **receiver transfer function** relating the pressure to the resulting electrical signal, **T_2(f);**

$$Y(f) = X(f)T_1(f)T_2(f)\underbrace{\tau_1\tau_2}_{A_\tau}e^{-(\alpha(f)+j\beta(f))l}e^{-(\alpha_w(f)+j\beta_w(f))l_w}$$

$$l_w = l_{w1} + l_{w2}$$

- For the direct water path signal, it is also possible to write a similar expression:

$$Y_w(f) = X(f)T_1(f)T_2(f)e^{-(\alpha_w(f)+j\beta_w(f))l_t}$$



$$Y(f) = Y_w(f) A_\tau e^{-(\alpha(f)+j\beta(f))l} e^{(\alpha_w(f)+j\beta_w(f))l}$$

$$Y(f) = Y_w(f) A_\tau e^{-[(\alpha(f)-\alpha_w(f))l + j(\beta(f)-\beta_w(f))l]}$$

Extending this rationale to multilayered objects,

$$\mathrm{Y(f)} = \mathrm{Y_w(f)} \mathrm{A}_\tau \mathrm{e}^{-\int_0^l (\alpha(\mathrm{x,f})-\alpha_\mathrm{w}(\mathrm{f}))\mathrm{dx} - \mathrm{j}\int_0^l (\beta(\mathrm{x,f})-\beta_\mathrm{w}(\mathrm{x,f}))\mathrm{dx}}$$

Attenuation in water is negligible, i.e, $\alpha_w(f) \approx 0$

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$$\beta(x, f) = \frac{2\pi f}{c(x)}$$

$$\beta_w(x, f) = \frac{2\pi f}{c_w}$$

$$\beta(x, f) - \beta_w(x, f) = \frac{2\pi f}{c_w}\left(\frac{c_w}{c(x)} - 1\right)$$

$$= \frac{2\pi f}{c_w}\left(\eta(x) - 1\right)$$

Refraction index

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$$T_d = \frac{1}{c_w}\int_0^l (\eta(x)-1)dx$$

$$Y(f) = A_\tau \underbrace{Y_w(f)e^{-\int_0^l \alpha(x,f)dx}}_{Y_w^{'}} e^{-j2\pi f T_d}$$

The corresponding signal can be obtained by taking the Inverse Fourier Transform:

$$y_w^{'}(t-T_d)$$

Attenuated water path signal

(It is a hypothetical signal that would be received if it underwent the same loss as the actual signal going through tissue.)

Reconstructing the attenuation coefficient α(x,y)

- For soft tissues the coefficient A_τ is negligible. The time delay in the measured signal may not be taken into account. Thus a line integral about the attenuation coefficient can be obtained from the amplitudes of the water path signal and the signal transmitted from the object :

$$\frac{y_w}{y} = \int_0^l \alpha(x, f)dx$$

- The same approach can be applied for different view angles and projection data can be obtained for each view.

- The reconstruction algorithms established for x-ray computerized tomography can be used to reconstruct α(x,y).

Ultrasonic Reflection Tomography

- Here the aim is to make cross sectional images for refractive index coefficient of the soft tissue. Remember the expression about the time delay of the wave propagating in x direction:

$$T_d = \frac{1}{c_w} \int_0^l (\eta(x) - 1)\, dx$$

- This can be generalized for waves propagating in any direction. Thus measurement of T_d provides a ray integral (projection data) of $\eta(x,y)$-1 for the corresponding view angle.

- Well known image reconstruction algorithms can be used to reconstruct $\eta(x,y)$ from time delay measurements.

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